

Homework 7

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Problem 17.1

Context: John has been hired to design an exciting carnival ride. Tiff, the carnival owner, has decided to create the world's greatest ferris wheel. Tiff isn't into math; she simply has a vision and has told John these constraints on her dream: (i) the wheel should rotate counterclockwise with an angular speed of 12 RPM; (ii) the linear speed of a rider should be 200 mph; (iii) the lowest point on the ride should be 4 feet above the level ground. Recall, we worked on this in Exercise 16.5.

Part A Problem: Impose a coordinate system and find the coordinates $T(t) = (x(t), y(t))$ of Tiff at time t seconds after she starts the ride.

Part A Solution: The Ferris wheel makes 12 rotations in a minute, or 720 in an hour. A rider sitting on the circumference of the wheel travels 200 miles per hour. Therefore, every rotation, the rider travels $\frac{200}{720}$ miles, or $\frac{200}{720} \cdot 5280$ feet. This is the circumference of the circle representing the wheel. We can find the radius r such that $2\pi r = \frac{200}{720} \cdot 5280$; r evaluates to $233.42724 \approx 233.42724$. For the ground to coincide with the x -axis, we will position the center of the wheel at $(0, 237.42724)$.

The Ferris wheel moves $\frac{1}{200} \cdot t \cdot 2\pi = \frac{2\pi t}{5}$ radians away from its original location at time t seconds. Looking at the 16.5 answer, Tiff begins at -0.06283 radians. Therefore, the x -location and the y -location can be described by:

$$\begin{aligned}x(t) &= 233.42724 \cos\left(\frac{2\pi t}{5} - 0.06283\right) \\y(t) &= 233.42724 \cos\left(\frac{2\pi t}{5} - 0.06283\right) + 237.42724\end{aligned}$$

Part B Problem: Tiff becomes a human missile after 6 seconds on the ride. Find Tiff's coordinates the instant she becomes a human missile.

Part B Solution: Plugging in $t = 6$ yields the coordinate $(85.92989, 454.46255)$.

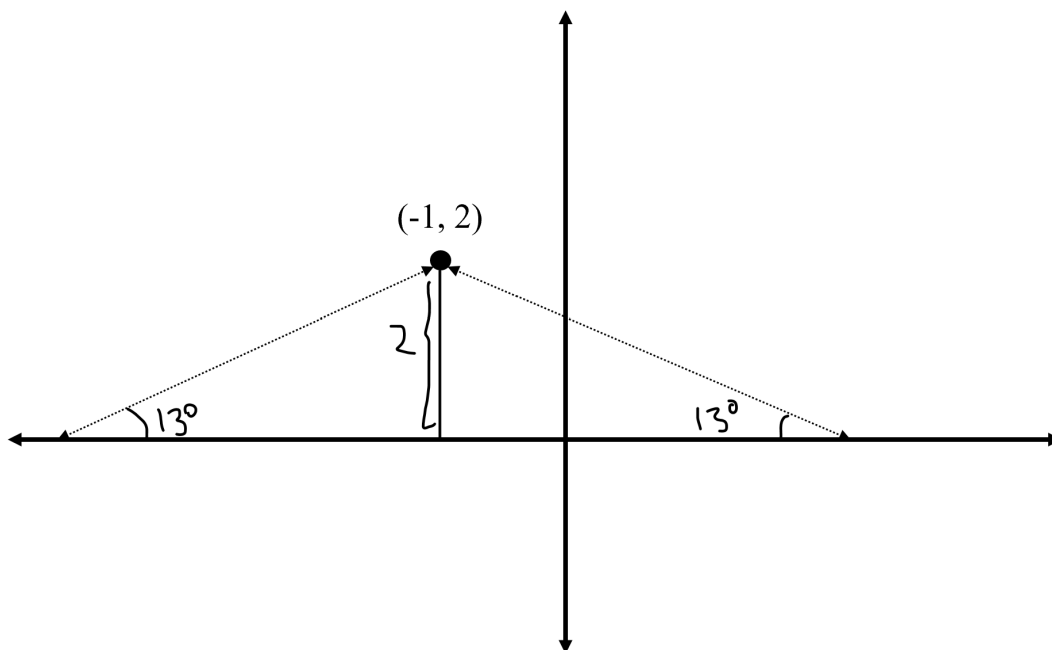
Part C Problem: Find the equation of the tangential line along which Tiff travels the instant she becomes a human missile. Sketch a picture indicating this line and her initial direction of motion along it when the seat detaches.

Part C Solution:

Problem 17.2

Part A Problem: Find the equation of a line passing through the point $(-1, 2)$ and making an angle of 13 degrees with the x -axis. (Note: There are two answers; find them both.)

Part A Solution: We have the following arrangement:



Let us first find the point at which the left right triangle intersects the x -axis.

$$\tan(13^\circ) = \frac{\text{opp}}{\text{adj}} = \frac{2}{\text{adj}} \implies \text{adj} = \frac{2}{\tan(13^\circ)} \approx 8.66295$$

This means that lines passing through $(-1, 2)$ that make a 13-degree angle with the x -axis must also pass either through $(-9.66295, 0)$ or $(7.66295, 0)$. Finding the equations for these two lines:

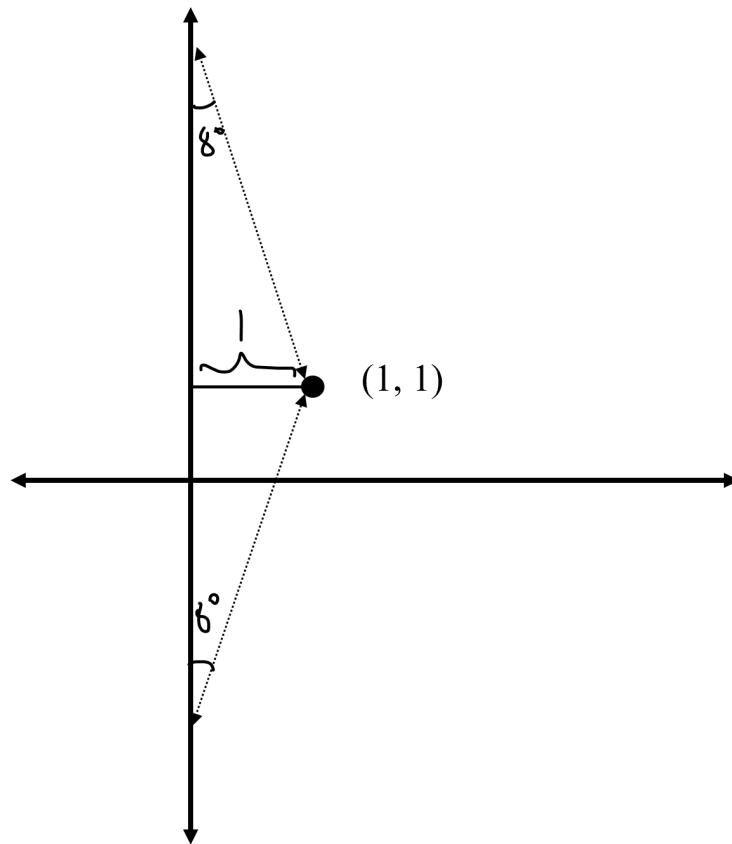
$$y = \frac{2}{-1 - (-9.66295)}x + b \implies y = 0.23086x + b; 0 = 0.23086(-9.66295) + b \implies b = 2.230788$$

$$y = \frac{2}{-1 - (7.66295)}x + b \implies y = -0.23086x + b; 0 = -0.23086(7.66295) + b \implies b = 1.76906$$

Thus, the equations of two lines passing through the point $(-1, 2)$ and making an angle of 13 degrees with the x -axis are $y = 0.23086x + 2.23078$ and $y = -0.23086x + 1.76906$

Part B Problem: Find the equation of a line making an angle of 8 degrees with the y -axis and passing through the point $(1, 1)$. (Note: There are two answers; find them both.)

Part B Solution: We have the following arrangement:



Let us find the point where the top line intersects the y -axis.

$$\tan(8^\circ) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\text{adj}} \implies \text{adj} = \frac{1}{\tan(8^\circ)} \approx 7.11536$$

This means that lines passing through $(1, 1)$ that make a 8-degree angle with the y -axis must pass either through $(0, 8.711536)$ or $(0, -6.11536)$. The slopes from $(1, 1)$ to these points are $-\frac{7.11536}{1}$ and $\frac{7.11536}{1}$, respectively. Two lines that satisfy the constraints outlined in the problem are thus $y = \pm 7.11536(x - 1) + 1$.

Problem 17.3

Problem: The crew of a helicopter needs to land temporarily in a forest and spot a flat horizontal piece of ground (a clearing in the forest) as a potential landing site, but are uncertain whether it is wide enough. They make two measurements from A (see picture) finding $\alpha = 25^\circ$ and $\beta = 54^\circ$. They rise vertically 100 feet to B and measure $\gamma = 47^\circ$. Determine the width of the clearing to the nearest foot.

Solution: Let h be the helicopter crew's initial height (before they rise 100 feet). Moreover, as the helicopter's horizontal position remains horizontal, let c_b and c_e denote the horizontal distance of the beginning and end of the clearing (from left-to-right) to the helicopter. From the diagram, we have the following relationships:

$$\begin{aligned}\tan(\beta) &= \frac{c_e}{h} \\ \tan(\beta - \alpha) &= \frac{c_b}{h} \\ \tan(\gamma) &= \frac{c_e}{h + 100}\end{aligned}$$

Filling in known values of α , β , and γ and evaluating:

$$\begin{aligned}\tan(54^\circ) \approx 1.37638 &= \frac{c_e}{h} \implies h = \frac{c_e}{1.37638} \\ \tan(54^\circ - 25^\circ) = \tan(29^\circ) \approx 0.55430 &= \frac{c_b}{h} \implies h = \frac{c_b}{0.55430}\end{aligned}$$

$$\tan(47^\circ) \approx 1.07236 = \frac{c_e}{h + 100}$$

It follows that $1.07236 = \frac{c_e}{\frac{c_e}{1.37638} + 100}$; thus, $c_e \approx 485.48610$. This yields a height of $\frac{485.8610}{1.37638}$. Plugging this value of h yields $c_b \approx 195.66743$. The length of the clearing is $c_e - c_b$, or $485.48610 - 195.66743 \approx 289.81867$. Rounded to the nearest, foot the clearing is **290 feet long**.

Problem 17.4

Context: Marla is running clockwise around a circular track. She runs at a constant speed of 3 meters per second. She takes 46 seconds to complete one lap of the track. From her starting point, it takes her 12 seconds to reach the northernmost point of the track. Impose a coordinate system with the center of the track at the origin, and the northernmost point on the positive y-axis.

Part A Problem: Give Marla's coordinates at her starting point.

Part A Solution: It takes Marla 46 seconds to complete one lap of the track, and she begins 12 seconds from the northernmost point of the track. The radius of the track itself can be derived from its circumference, which is $46 \times 3 = 138$ meters, yielding $2\pi r = 138 \implies r = \frac{138}{2\pi} \approx 21.96338$ meters.

Every second, Marla travels $\frac{-3(t)}{46 \cdot 3} \cdot 2\pi$ radians farther from her original location. We could thus model Marla's x -location as $\frac{138}{2\pi} \cos\left(\frac{-3(t)}{46 \cdot 3} \cdot 2\pi\right)$. Marla begins at the location with the angle $\frac{\pi}{2} + 2\pi \cdot \frac{12}{46}$ radians, though, so the previous model is incorrect. To account for this offset, we shift the co/sinusoidal function left by whatever angle Marla begins at. The models for Marla's x and y locations are thus:

$$x(t) = \frac{138}{2\pi} \cos\left(\frac{-3(t)}{46 \cdot 3} \cdot 2\pi + \left(\frac{\pi}{2} + 2\pi \cdot \frac{12}{46}\right)\right) \quad y(t) = \frac{138}{2\pi} \sin\left(\frac{-3(t)}{46 \cdot 3} \cdot 2\pi + \left(\frac{\pi}{2} + 2\pi \cdot \frac{12}{46}\right)\right)$$

With $t = 0$, we have **$(-21.91218, -1.49883)$** .

Part B Problem: Give Marla's coordinates when she has been running for 10 seconds.

Part B Solution: Plugging $t = 10$ yields **$(-5.92564, 21.14892)$** .

Part B Problem: Give Marla's coordinates when she has been running for 901.3 seconds.

Part B Solution: Plugging $t = 901.3$ yields **$(19.07064, -10.89497)$** .

Problem 17.5

Part A Problem: If $\theta = 34^\circ$, then what are your xy -coordinates after 4 minutes?

Part A Solution: To begin, we know that the Ferris wheel is rotating such that a degree of $\frac{72t}{24} \cdot 2\pi$ radians is formed from its original location at time t , in minutes. All that is needed is to adjust the movement to the central position by shifting the curves back by however much they have been shifted. In this case, we have:

$$x(t) = 24 \cos\left(\frac{72(4)}{24} \cdot 2\pi + 34 \cdot \frac{2\pi}{360}\right) = \mathbf{19.8969}$$

$$y(t) = 24 \sin\left(\frac{72(4)}{24} \cdot 2\pi + 34 \cdot \frac{2\pi}{360}\right) = \mathbf{13.42062}$$

Part B Problem: If $\theta = 20^\circ$, then what are your xy -coordinates after 45 minutes?

Part B Solution: Using the same logic, we have:

$$x(t) = 24 \cos\left(\frac{72(45)}{24} \cdot 2\pi + 20 \cdot \frac{2\pi}{360}\right) = \mathbf{22.55262}$$

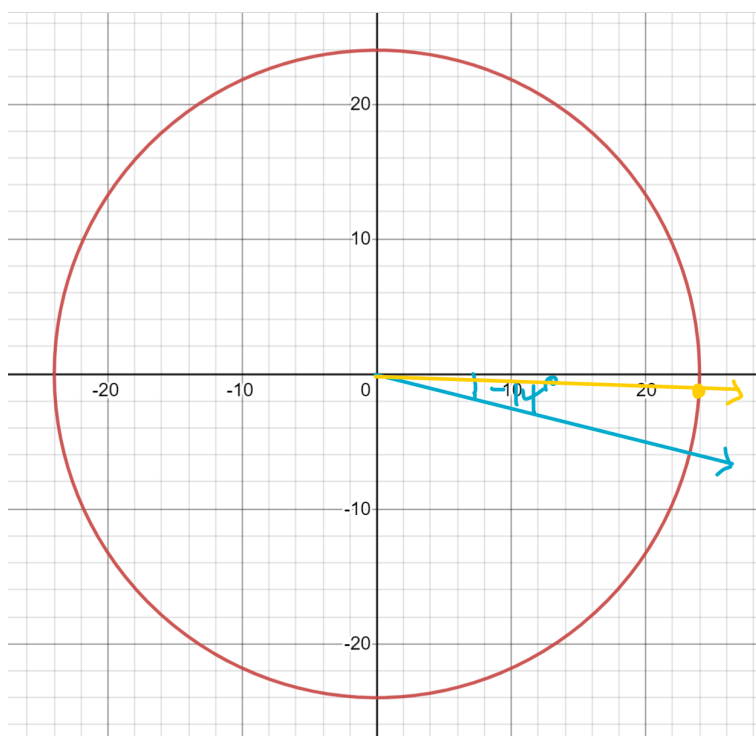
$$y(t) = 24 \sin\left(\frac{72(45)}{24} \cdot 2\pi + 20 \cdot \frac{2\pi}{360}\right) = \mathbf{8.20848}$$

Part C Problem: If $\theta = -14^\circ$, then what are your xy -coordinates after 6 seconds? Draw an accurate picture of the situation.

Part C Solution: Using the same logic, but in this case shifting the curves right by subtracting rather than adding the correction angle, we have:

$$x(t) = 24 \cos \left(\frac{72 \left(\frac{6}{60} \right)}{24} \cdot 2\pi - 14 \cdot \frac{2\pi}{360} \right) = -1.67415$$

$$y(t) = 24 \sin \left(\frac{72 \left(\frac{6}{60} \right)}{24} \cdot 2\pi - 14 \cdot \frac{2\pi}{360} \right) = 23.94153$$

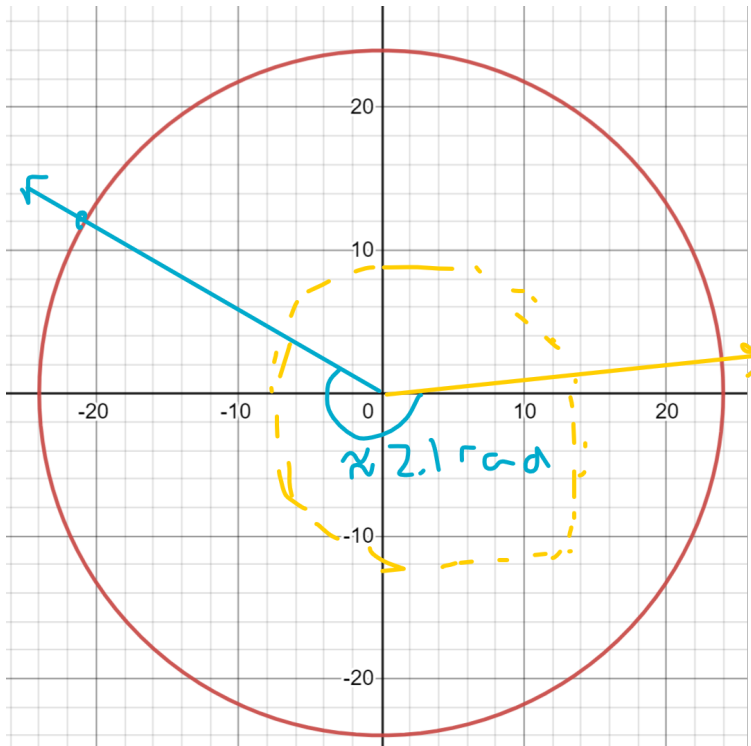


Part D Problem: If $\theta = -2.1$ rad, then what are your xy -coordinates after 2 hours and 7 seconds? Draw an accurate picture of the situation.

Part D Solution: Using the same logic, we have:

$$x(t) = 24 \cos \left(\frac{72 \left(2 + \frac{7}{60} \right)}{24} \cdot 2\pi - 2.1 \right) = 23.882$$

$$y(t) = 24 \sin \left(\frac{72 \left(2 + \frac{7}{60} \right)}{24} \cdot 2\pi - 2.1 \right) = 23.94153$$

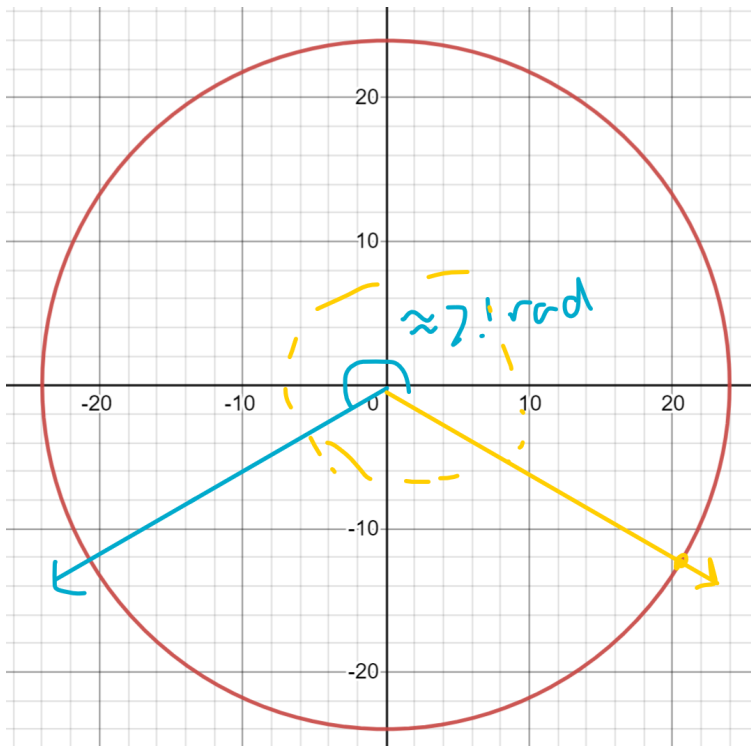


Part E Problem: If $\theta = -2.1$ rad, then what are your xy -coordinates after 2 hours and 7 seconds? Draw an accurate picture of the situation.

Part E Solution: Using the same logic, we have:

$$x(t) = 24 \cos \left(\frac{72 \left(\frac{5}{60} \right)}{24} \cdot 2\pi + 2.1 \right) = -20.71702$$

$$y(t) = 24 \sin \left(\frac{72 \left(\frac{5}{60} \right)}{24} \cdot 2\pi + 2.1 \right) = -12.11630$$



Additional Problem

Problem: Look at the images below (A and B). Each one depicts a box in perspective, with the horizon drawn in gray. For each one, suppose that the height of the box is 1, and that it is flat on the ground. What are the other two dimensions of each one? If each image is displayed as an image 1 unit across, how close should the viewer be so that the perspective looks accurate?

Solution: We can begin by approximating the lengths of the cube in terms of two-dimensional representation. Out of convenience, it is nice to have the lengths be between -1 and 1 , so each length can be divided by 5 for convenience.

