Collingwood Homework 8

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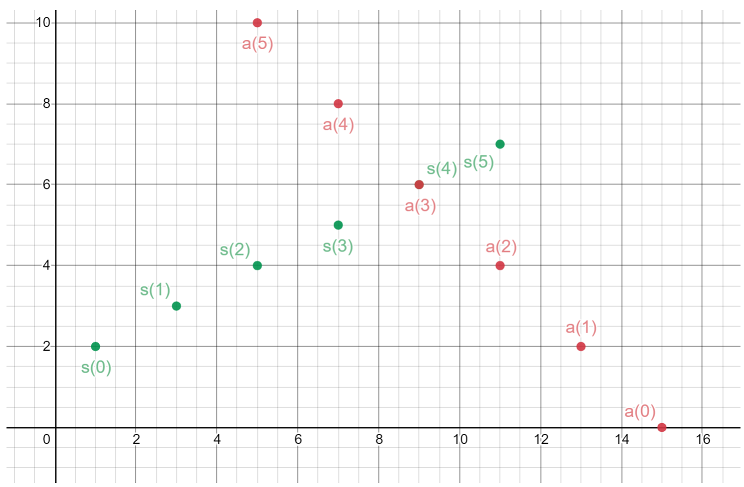
2.8 Context: A spider is located at the position (1,2) in a coordinate system, where the units on each axis are feet. An ant is located at the position (15,0) in the same coordinate system. Assume the location of the spider after t minutes is $s(t) = (1+2t, 2+t)$ and the location of the ant after t minutes is $a(t) = (15-2t, 2t)$.

2.8a Problem: Sketch a picture of the situation, indicating the locations of the spider and ant at times t = 0, 1, 2, 3, 4, 5 minutes. Label the locations of the bugs in your picture, using the notation $s(0), s(1), ..., s(5)$, $a(0), a(1), ..., a(5)$.

2.8a Solution: Graphed in Desmos.



2.8b Problem: When will the x-coordinate of the spider equal 5? When will the y-coordinate of the ant equal 5?

2.8b Solution: Looking at the diagram, at $\boxed{t = 2}$ the spider's x-coordinate equals 5. In order to find the time at which the y-coordinate of the ant equals 5, we can write a linear equation $5 = 2t$, where $2t$ is the y-location of the ant at time t. Thus, at time $\boxed{t = \frac{5}{2}}$ the y-coordinate of the ant equals 5.

2.8c Problem: Where is the spider located when its y-coordinate is 3?

2.8c Solution: Looking at the coordinates, the Spider is located at point $\boxed{(3,3)}$.

2.8d Problem: Where is each bug located when the y-coordinate of the spider is twice as large as the y-coordinate of the ant?

2.8d Solution: The following equation can be set up to model this, where $2 + t$ is the y-coordinate of the spider and $2t$ is the y-coordinate of the ant.

$$\begin{aligned}2+t &= 2(2t)\\ 2+t &= 4t\\ 2 &= 3t\\ \tfrac{2}{3} &= t\end{aligned}$$

Plugging this into the two given equations for location of the spider and the ant at time t, we derive:

- Spider located at $s(\frac{2}{3}) = (1 + 2 \cdot \frac{2}{3}, 2 + \frac{2}{3}) = (\frac{7}{3}, \frac{8}{3})$.
- Ant located at $a(\frac{2}{3}) = (15 - 2 \cdot \frac{2}{3}, 2 \cdot \frac{2}{3}) = (\frac{41}{3}, \frac{4}{3})$.

Therefore, the locations of the Spider and the Ant are $\boxed{(\frac{7}{3}, \frac{8}{3}) \text{ and } (\frac{41}{3}, \frac{4}{3})}$, respectively.

2.8e Problem: How far apart are the bugs when their x-coordinates coincide? Draw a picture, indicating the locations of each bug when their x-coordinates coincide.

2.8e Solution: To find the time t at which the bugs' x-coordinates coincide, we can set the equations for x-coordinate at time t for the ant and the spider:

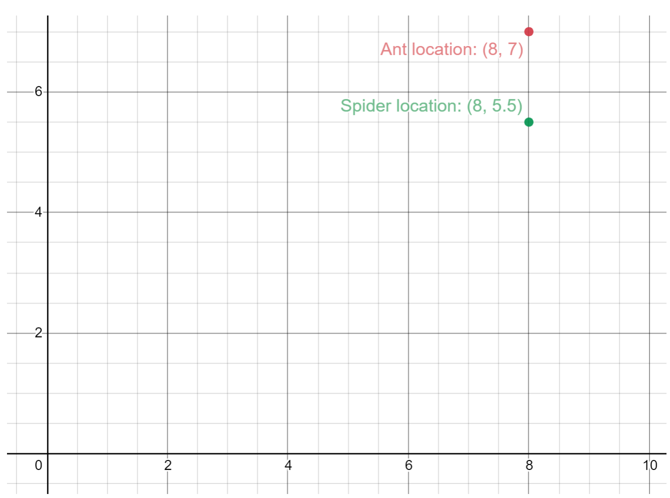
$$\begin{aligned}1+2t &= 15-2t\\ 4t &= 14\\ t &= \tfrac{7}{2}\end{aligned}$$

At time $t = \frac{7}{2}$, the y coordinates of the spider and the ant are:

$$2 + t \to 2 + \tfrac{7}{2} \to \tfrac{11}{2}$$

$$2t \to 2\left(\tfrac{7}{2}\right) \to 7$$

The difference between the y-values is $\frac{14}{2} - \frac{11}{2} = \frac{3}{2} = 1.5$. There is no need to account for changes in the x-coordinate because the change is 0. Therefore, the two are $\boxed{\text{1.5 feet apart}}$ when their x-coordinates coincide.



2.8f Problem: A sugar cube is located at the position (9,6). Explain why each bug will pass through the position of the sugar cube. Which bug reaches the sugar cube first?

2.8f Solution: Since the equations for the components x_a, x_s, y_a, and y_s (Where subscript-a denotes the ant's location and subscript-s denotes the spider's location) are given, one can solve for the time they pass through (9,6), if at all.

$$\begin{aligned} x_a &= 9 \\ 15 - 2t &= 9 \\ -2t &= -6 \\ t &= 3 \end{aligned}$$

If the ant also passed through the y-coordinate 6 at $t = 3$ minutes, then the ant passes through the point (9, 6).

$$\begin{aligned} y_a &= 6 \\ 2t &= 6 \\ t &= 3 \end{aligned}$$

The ant reaches the cube in $\boxed{\text{3 minutes}}$.

$$\begin{aligned} x_s &= 9 \\ 1 + 2t &= 9 \\ 2t &= 8 \\ t &= 4 \end{aligned}$$

If the spider also passed through the y-coordinate 6 at $t = 4$ minutes, then the spider passes through the point (9, 6).

$$\begin{aligned} y_s &= 6 \\ 2 + t &= 6 \\ t &= 4 \end{aligned}$$

The spider reaches the cube in $\boxed{\text{4 minutes}}$. Therefore, $\boxed{\text{the ant reaches the sugar cube first.}}$

2.8g Problem: Find the speed of each bug along its line of motion; which bug is moving faster?

2.8g Solution: According to the equations of the ant and the spider,

- The spider's location at time t is $(1 + 2t, 2 + t)$. Every increment in t, the spider moves 2 feet right and 1 feet up. Therefore, it moves $\sqrt{2^2 + 1^2} = \sqrt{5}$ feet per minute.
- The ant's location at time $(15 - 2t, 2t)$. Every increment in t, the ant moves 2 feet to the left and 2 feet up. Therefore, it moves $\sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$ feet per minute.

Therefore, the spider moves $\boxed{\sqrt{5}\text{ feet per minute}}$ and the ant moves $\boxed{2\sqrt{2}\text{ feet per minute}}$. Since $2\sqrt{2} > \sqrt{5}$, the $\boxed{\text{ant is moving faster.}}$

2.9 Problem: A Ferrari is heading south at a constant speed on Broadway (a north/south street) at the same time a Mercedes is heading west on Aloha Avenue (an east/west street). The Ferrari is 624 feet north of the intersection of Broadway and Aloha, at the same time that the Mercedes is 400 feet east of the intersection. Assume the Mercedes is traveling at the constant speed of 32 miles/hour. Find the speed of the Ferrari so that a collision occurs in the intersection of Broadway and Aloha.

2.9 Solution: The Ferrari is 624 feet from the intersection and travelling at a speed of s_f mph. The Mercedes is 400 feet from the intersection and travelling at a speed of 32 mph. For there to be a collision, both must travel the remainder of the distance from the intersection in the same amount of time. Fitting the data into a $d = rt$ framework:

$$624 \text{ feet} = s_f \text{ mph} \cdot t \text{ hours}$$

$$400 \text{ feet} = 32 \text{ mph} \cdot t \text{ hours}$$

Converting units:

$$\tfrac{624}{5280} = \tfrac{13}{110} \text{ miles} = s_f \text{ mph} \cdot t \text{ hours}$$
$$\tfrac{400}{5280} = \tfrac{5}{66} \text{ miles} = 32 \text{ mph} \cdot t \text{ hours}$$

Solving the second equation for t:

$$\tfrac{5}{66} = 32 \cdot t$$

$$\tfrac{5}{2112} = t$$

Plugging this value of t into the first equation to solve for s_f:

$$\tfrac{13}{110} = s_f \cdot \tfrac{5}{2112}$$
$$\tfrac{1248}{25} = 49.92 = s_f$$

Therefore, the Ferrari must travel at $\boxed{49.92 \text{ mph}}$.