Collingwood Homework 7

Andre Ye

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2.6 Context: Allyson and Adrian have decided to connect their ankles with a bungee cord; one end is tied to each person's ankle. The cord is 30 feet long, but can stretch up to 90 feet. They both start from the same location. Allyson moves 10 ft/sec and Adrian moves 8 ft/sec in the directions indicated.

2.6a Question: Where are the two girls located after two seconds?

2.6a Solution: Let (0,0) be the original starting location and one unit distance represent one foot. Then, Adrian moves towards the negative side of the x-axis, and Allyson moves towards the positive side of the y-axis. Since Adrian moves at 8 ft/sec, she moves 16 feet in two seconds; applying the same reasoning, Allyson moves 20 feet in two seconds. Applying these changes in their respective directions, $\boxed{\text{Adrian is at location (-16,0) and Allyson is at location (0,20)}}$.

2.6b Question: After 2 seconds, will the slack in the bungee cord be used up?

2.6b Solution: The two girls are at location (-16,0) and (0,20). Their distance, then, is $\sqrt{16^2+20^2} = \sqrt{656} \approx 25.61$ feet, which is less than the cord's length (30 feet). Therefore, after 2 seconds, the slack in the bungee cord $\boxed{\text{will not be used up.}}$

2.6c Question: Determine when the bungee cord first becomes tight; i.e. there is no slack in the line. Where are the girls located when this occurs?

2.6c Solution: Representing the location of the girls in terms of time t in seconds, where x_d and y_d represent Adrian's location, and x_l and y_l represent Allyson's location, we have $x_d = -8t$, $y_d = 0$, $x_l = 0$, $y_l = 10t$. We want to find the time t in which the distance between the two reaches 30 feet, at which there will be no more slack in the bungee cord.

$$\begin{aligned}
30 &= \sqrt{(0-(-8t))^2 + (10t-0)^2} \\
900 &= 64t^2 + 100t^2 \\
900 &= 164t^2 \\
\frac{225}{41} &= t^2 \\
\sqrt{\frac{225}{41}} &= t
\end{aligned}$$

Therefore, it takes $\boxed{t \approx 2.34 \text{ seconds}}$ for the bungee cord first to become tight.

2.6d Question: When will the bungee cord first touch the corner of the building?

2.6d Solution: The bottom-left corner of the building is located at point (-20, 30). The line the bungee follows can be represented as $y = \frac{5}{4}x + b$, since for every four feet to the left Adrian moves, Allyson moves five feet up. Plugging (-20, 30) into the equation yields $30 = -25 + b \to b = 55$, which means that the y-intercept is (0,55). Hence, the bungee cord touches the building when Allyson is at 55 feet. Since Allyson moves 10 feet per second, this happens at $\frac{55}{10} = 5.5$ seconds. Therefore, it takes $\boxed{5.5 \text{ seconds}}$ for the bungee cord to first touch the corner of the building.

2.7 Context: Brooke is located 5 miles out from the nearest point A along a straight shoreline in her seakayak. Hunger strikes and she wants to make it to Kono's for lunch; see picture. Brooke can paddle 2 mph and walk 4 mph.

2.7a Question: If she paddles along a straight line course to the shore, find an expression that computes the total time to reach lunch in terms of the location where Brooke beaches the boat.

2.7a Solution: Let the shore be the x-axis, and let Brooke's original location be on he y-axis. Let x_k be the x-coordinate location at which Brooke's kayak reaches shore. There are two distance components that need to be accounted for: the distance from her original location to the shore, and the distance from her shore-boarding location to Kono's.

- Brooke begins at (0,5) and she kayaks in a straight line to (x,0). Therefore, the distance between the two points is $\sqrt{25+x^2}$. Brooke paddles at 2 miles per hour, meaning she takes one hour to paddle two miles; therefore the time for her to paddle to (x,0) is $\frac{1}{2}\sqrt{25+x^2}$.
- Once Brooke reaches shore at (x,0), she travels to (6,0). Hence, the distance is $6-x$ miles. Brooke walks at 4 miles per hour, meaning she takes one hour to walk four miles; therefore the time for her to walk from (x,0) to (6,0) is $\frac{1}{4}(6-x)$.

Adding the two times, the total time is $\boxed{\frac{1}{2}\sqrt{25+x^2}+\frac{1}{4}(6-x) \text{ hours}}$.

2.7b Question: Determine the total time to reach Kono's if she paddles directly to the point "A".

2.7b Solution: At this point, $x = 0$. $\frac{1}{2}\sqrt{25+0^2}+\frac{1}{4}(6-0) = \frac{5}{2}+\frac{3}{2} = 4$. Therefore, if Brooke paddles directly to point "A", she takes $\boxed{4 \text{ hours}}$ to reach Kono's.

2.7c Question: Determine the total time to reach Kono's if she paddles directly to Kono's.

2.7c Solution: At this point, $x = 6$. $\frac{1}{2}\sqrt{25+6^2}+\frac{1}{4}(6-6) = \frac{\sqrt{61}}{2} \approx 3.91$ hours. Therefore, if Brooke paddles directly to point Kono's, she takes $\boxed{\approx 3.9 \text{ hours}}$ to reach Kono's.

2.7d Question: Do you think your answer to (b) or (c) is the minimum time required for Brooke to reach lunch?

2.7d Solution: Let us calculate the time taken for $x = 3$ - some time in between the two extremes presented in (b) and (c) - to see if the two are minimums or not. $\frac{1}{2}\sqrt{25+3^2}+\frac{1}{4}(6-3) = \frac{\sqrt{34}}{2}+\frac{3}{4} \approx 3.67$ hours. This is less than the time for both alternatives; therefore, $\boxed{\text{neither answers for (b) nor (c)}}$ are the minimum time required for Brooke to reach lunch.