Collingwood Homework 6

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2.3 Problem: Steve and Elsie are camping in the desert, but have decided to part ways. Steve heads North, at 6 AM, and walks steadily at 3 miles per hour. Elsie sleeps in, and starts walking West at 3.5 miles per hour starting at 8 AM. When will the distance between them be 25 miles?

2.3 Solution: Let d_s be the distance north that Steve has walked, and let d_e be the distance west Elsie has walked. Using the $d = rt$ framework, Steve's distance can be represented as $d_s = 3t$, where t is the number of hours from 6 AM Steve has been walking. Similarly, Elsie's distance is $d_e = 3.5(t-2)$, since Elsie begins walking two hours after Steve begins.

The distance between the two is given by $d_t = \sqrt{(d_s)^2 + (d_e)^2}$, since d_s and d_e represent the lengths of the vertical and horizontal components of a right triangle.

To find the time it takes for the distance between them to reach 25 miles, the following equation is true:

$$\begin{aligned} 25 &= \sqrt{(d_s)^2 + (d_e)^2} \\ 25 &= \sqrt{(3t)^2 + (3.5(t-2))^2} \\ 25^2 &= 9t^2 + (3.5t - 7)^2 \\ 625 &= 9t^2 + 12.25t^2 - 49t + 49 \\ 0 &= 21.25t^2 - 49t - 576 \end{aligned}$$

Using the quadratic formula, we can find the value of t:

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{49 \pm \sqrt{(-49)^2 - 4(21.25)(-576)}}{2(21.25)} \\ &= \frac{49 \pm \sqrt{51361}}{42.5} \end{aligned}$$

Thus, the two solutions are $\frac{49+\sqrt{51361}}{42.5} \approx 6.4854$ hours and $\frac{49-\sqrt{51361}}{42.5} \approx -4.1795$ hours. Since hours cannot be negative, it takes Steve and Elsie ≈ 6.4854 hours to be 25 miles apart. This is about 6 hours and $0.4854 \cdot 60 = 29$ minutes after 6 AM, which means it will be $\boxed{\approx\text{12:29 PM that afternoon}}$.

2.4 Context: Erik's disabled sailboat is floating at a stationary location 3 miles East and 2 miles North of Kingston. A ferry leaves Kingston heading due East toward Edmonds at 12 mph. At the same time, Erik leaves the sailboat in a dinghy heading due South at 10 ft/sec (hoping to intercept the ferry). Edmonds is 6 miles due East of Kingston.

2.4a Problem: Compute Erik's speed in mph and the Ferry speed in ft/sec.

2.4a Solution: Erik travels at 10 ft/sec. Because there are 60 seconds in a minute$\times$60 minutes in an hour = 3600 seconds in an hour, he travels 36000 ft/hour. Because there are 5280 feet in a mile, he travels $\frac{36000}{5280} = \frac{75}{11}$

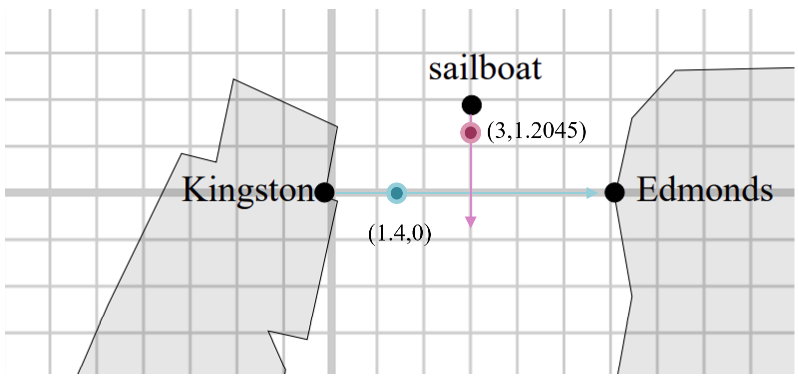
miles/hour. The ferry travels at 12 mph. Because there are 5280 feet in a mile, the ferry travels $12 \cdot 5280 =$ 63360 feet per hour. Because there are 3600 seconds in an hour, the ferry travels $\frac{63360}{3600} = \frac{88}{5}$ feet per second. Hence, Erik's speed is $\boxed{\frac{75}{11} \approx 6.81 \text{ miles/hour}}$ and the ferry's speed is $\boxed{\frac{88}{5} = 17.6 \text{ ft/sec}}$.

2.4b Problem: Impose a coordinate system and complete this table of data concerning locations (i.e., coordinates) of Erik and the ferry. Insert into the picture the locations of the ferry and Erik after 7 minutes.

2.4b Solution: Let the ferry begin at (0,0) (Kingston). Hence, because Erik is 3 miles East and 2 miles North of Kingston, Erik begins at (3,2). Because Edmonds is 6 miles East of Kingston, it is located at (6,0).

- At 0 sec., all objects are at their original locations. The distance is then $\sqrt{13} \approx 3.6056$ miles.
- At 30 sec., the ferry, which travels 17.6 feet per second, is $30 \cdot 17.6 = 528$ feet, or $\frac{528}{5280} = 0.1$ miles, East of Kingston at point (0.1,0). At thirty sec., Erik, who moves at 10 feet per second, is $30 \cdot 10 =$ 300 feet, or $\frac{300}{5280} \approx 0.0568$ miles South of his starting point at (3,1.9432). The distance, then, is $\approx \sqrt{(3-0.1)^2 + (1.9432-0)^2} \approx 3.4908$ miles.
- At 7 minutes, the ferry, which travels at 12 miles per hour, is $\frac{7}{60} \cdot 12 = 1.4$ miles East of Kingston, at location (1.4,0). At 7 minutes, Erik, who travels at 10 ft/sec, is $7 \cdot 60 \cdot 10 = 4200$ feet, or $\frac{4200}{5280} \approx 0.7955$ miles South of his starting point at location (3, 1.2045). The distance, then, is $\approx \sqrt{(3-1.4)^2 + (1.2045-0)^2} \approx 2.0027$ miles.

  As requested by the problem, a visualization of the points at 7 minutes:



- At t hours, the ferry, which travels at 12 miles per hour is $12t$ miles East of Kingston (0,0), which means it is at location $(12t,0)$. At t hours, Erik, who travels at $\frac{75}{11}$ miles per hour South from his starting position at (3, 2), which means he ends at $(3, 2-\frac{75}{11}t)$. Using the distance formula, the distance between the two points is $d = \sqrt{(3-12t)^2 + (2-\frac{75}{11}t)^2}$.

Time	Ferry	Erik	Distance Between
0 sec	(0,0)	(3,2)	3.6056 miles
30 sec	(0.1,0)	(3,1.9432)	3.4908 miles
7 min	(1.4,0)	(3, 1.2045)	2.0027 miles
t hours	$(12t,0)$	$(3, 2-\frac{75}{11}t)$	$\sqrt{(3-12t)^2 + (2-\frac{75}{11}t)^2}$ miles

2.4c Problem: Explain why Erik misses the ferry.

2.4c Solution: Let x_f and y_f represent the x and y coordinate of the ferry, and let x_e and y_e represent the x and y coordinates of Erik. Then, the following relationships are true:

- $x_f = 12t$

- $y_f = 0$
- $x_e = 3$
- $y_e = 2 - \frac{75}{11}t$

If the ferry and Erik meet, there exists some time t such that the x and y coordinates of both entities are equal.

$$x_f = x_e \rightarrow 12t = 3 \rightarrow t = \frac{1}{4}$$
$$y_f = y_e \rightarrow 0 = 2 - \frac{75}{11}t \rightarrow t = \frac{22}{75}$$

The solutions for t are not equivalent; therefore, Erik misses the ferry.

2.4d Problem: After 10 minutes, a Coast Guard boat leaves Kingston heading due East at a speed of 25 ft/sec. Will the Coast Guard boat catch the ferry before it reaches Edmonds? Explain.

2.4d Solution: Let d_f represent the number of feet East the ferry is from Kingston; let d_c represent the number of feet East the Coast Guard boat is from Kingston; let t represent the number of seconds that pass. The following $d = rt$-based equations are true:

$$d_f = 25(t - 10 \cdot 60)$$
$$d_c = \tfrac{88}{5}t$$

If the Coast Guard boat can catch up to the ferry, then there exists a valid time t such that $d_f = d_c$.

$$\begin{aligned} d_f &= d_c \\ 25(t - 600) &= 17.6t \\ 25t - 15000 &= 17.6t \\ 7.4t &= 15000 \\ t &\approx 2027 \text{ seconds} \end{aligned}$$

The ferry takes 30 minutes $\cdot$ 60 seconds in a minute = 1800seconds to travel the 6 miles at 12 mph from Kingston to Edmonds. However, the Coast Guard requires 2027 seconds to catch up, which is an invalid time (larger than the allotted time) ; therefore $\boxed{\text{the Coast Guard cannot catch up to the ferry.}}$