Collingwood Homework 5

Andre Ye

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2.1 Context: In the following four cases, let P be the initial (starting) point and Q the ending point; recall Equation 2.2 and Figure 2.10 on Page 19. Compute $d =$ the distance from P to Q, Δx and Δy. Give your answer in exact form; eg. $\sqrt{2}$ is an exact answer, whereas 1.41 is an approximation of $\sqrt{2}$.

2.1 Note: In problems 2.1a - 2.1d, the following formulas are directly used to calculate d (the distance), Δx (the change in x), and Δy (the change in y).

- $d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$.
- $\Delta x =$ ending point $x -$ starting point x.
- $\Delta y =$ ending point $y -$ ending point y.

2.1a Question: $P = (0, 0)$, $Q = (1, 1)$.

2.1a Solution: $d = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{1+1} = \sqrt{2}$. $\Delta x = 1 - 0 = 1$. $\Delta y = 1 - 0 = 1$. Therefore, $\boxed{d = \sqrt{2}, \Delta x = 1, \Delta y = 1}$.

2.1b Question: $P = (2, 1), Q = (1, -1)$.

2.1b Solution: $d = \sqrt{(2-1)^2 + (1-(-1))^2} = \sqrt{1+4} = \sqrt{5}$. $\Delta x = 1 - 2 = -1$. $\Delta y = -1 - 1 = -2$. Therefore, $\boxed{d = \sqrt{5}, \Delta x = -1, \Delta y = -2}$.

2.1c Question: $P = (-1, 2), Q = (4, -1)$.

2.1c Solution: $d = \sqrt{(-1-4)^2 + (2-(-1))^2} = \sqrt{25+9} = \sqrt{34}$. $\Delta x = 4 - (-1) = 5$. $\Delta y = -1 - 2 = -3$. Therefore, $\boxed{d = \sqrt{34}, \Delta x = 5, \Delta y = -3}$.

2.1d Question: $P = (1, 2), Q = (1 + 3t, 3 + t)$.

2.1d Solution: $d = \sqrt{(1-(1+3t))^2 + (2-(3+t))^2} = \sqrt{(-3t)^2 + (-1-t)^2} = \sqrt{9t^2 + 1 + 2t + t^2} = \sqrt{10t^2 + 2t + 1}$. $\Delta x = (1+3t) - 1 = 3t$. $\Delta y = (3+t) - 2 = 1+t$. Therefore, $\boxed{d = \sqrt{10t^2 + 2t + 1}, \Delta x = 3t, \Delta y = 1 + t}$.

2.2 Context: Start with two points $M = (a, b)$ and $N = (s, t)$ in the xy-coordinate system. Let d be the distance between these two points. Answer these questions and make sure you can justify your answers:

2.2a Question: TRUE or FALSE: $d = \sqrt{(a-s)^2 + (b-t)^2}$.

2.1a Solution: The two points are (a, b) and (s, t). Thus, $\Delta x = a - s$ and $\Delta y = b - t$. Hence, the distance is $d = \sqrt{(a-s)^2 + (b-t)^2}$. Therefore, the statement is $\boxed{\text{TRUE}}$.

2.2b Question: TRUE or FALSE: $d = \sqrt{(a-s)^2 + (t-b)^2}$.

2.1b Solution: There generalized form of the distance formula: $d = \sqrt{(|x_1 - x_0|)^2 + (|y_1 - y_0|)^2}$. Because the squaring function x^2 automatically converts all negative values of x into positive ones, the absolute value is usually ignored. However, this means that $|a-b| = |b-a| \rightarrow (a-b)^2 = (b-a)^2$. Therefore, it does not matter which order the $(\Delta x)^2$ and $(\Delta y)^2$ components are in, and the statement is $\boxed{\text{TRUE}}$.

2.2c Question: TRUE or FALSE: $d = \sqrt{(s-a)^2 + (t-b)^2}$.

2.1c Solution: Using the same logic as above, as long as the two elements within the parenthesis are comparable (are both representing x or y of some point), the order in which they are written does not matter. Therefore, the statement is $\boxed{\text{TRUE}}$.

2.2d Question: Suppose M is the beginning point and N is the ending point; recall Equation 2.2 and Figure 2.10 on Page 19. What is Δx? What is Δy?

2.2d Solution: $\Delta x =$ ending point $x -$ starting point x and $\Delta y =$ ending point $y -$ ending point y. In this case, this is $\Delta x = N_x - M_x$ and $\Delta y = N_y - M_y$. Therefore, $\boxed{\Delta x = s - a \text{ and } \Delta y = t - b}$.

2.2e Question: Suppose N is the beginning point and M is the ending point; recall Equation 2.2 and Figure 2.10 on Page 19. What is Δx? What is Δy?

2.2e Solution: $\Delta x =$ ending point $x -$ starting point x and $\Delta y =$ ending point $y -$ ending point y. In this case, this is $\Delta x = M_x - N_x$ and $\Delta y = M_y - N_y$. Therefore, $\boxed{\Delta x = a - s \text{ and } \Delta y = b - t}$.

2.2f Question: If $\Delta x = 0$, what can you say about the relationship between the positions of the two points M and N ? If $\Delta y = 0$, what can you say about the relationship between the positions of the two points M and N? (Hint: Use some specific values for the coordinates and draw some pictures to see what is going on.)

2.2f Solution: When $\Delta x = 0$, the change in x when moving from one point to another is 0; this means that the two points are on the same vertical line ($x = q$, where q is some fixed value). When $\Delta y = 0$, the change in y when moving from one point to another is 0; this means that the two points are on the same horizontal line ($y = q$, where q is some fixed value).