Collingwood Homework 4

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1.11 Question: A typical cell in the human body contains molecules of deoxyribonucleic acid, referred to as *DNA* for short. In the cell, this DNA is all twisted together in a tight little packet. But imagine unwinding (straightening out) all of the DNA form a single typical cell and laying it "end-to-end"; then the sum total length will be approximately 2 meters. Assume the human body has 10^{14} cells containing DNA. How many times would the sum total length of DNA in your body wrap around the equator of the earth?

1.11 Solution: One cell contributes ≈ 2 meters in length; therefore, 10^{14} cells measure $10^{14} \cdot 2 = 2 \times 10^{14}$ meters in length. A Wikipedia search will yield that Earth's equator is $\approx$ 40,075 km long. Since there are 1000 meters in a kilometer, this is equivalent to $\approx 40,075 \cdot 1000 = 40,075,000 = 4.0075 \times 10^7$ meters. To find how many times the total DNA length could wrap around the equator, one divides the former by the latter:

$$\frac{2 \times 10^{14}}{4.0075 \times 10^7} = \frac{2 \times 10^7}{4.0075} = 4,990,642.545 \text{ times}$$

Hence, when stretched end-to-end, the total length of DNA in your body can wrap around the equator of the earth $\boxed{\text{4,9990,642.545} \approx 5 \times 10^6 \text{ times.}}$

1.12 Context: A water pipe mounted to the ceiling has a leak and is dripping onto the floor below, creating a circular puddle of water. The area of the circular puddle is increasing at a constant rate of 11 cm^2/hour.

1.12a Question: Find the area and radius of the puddle after 1 minute, 92 minutes, 5 hours, 1 day.

1.12a Solution: The area of a puddle increases at 11 cm^2 per hour; therefore, the area of the puddle after h hours is then $11h$ cm^2.

- At *1 minute*, $h = \frac{1}{60}$. Therefore, the area of the puddle is $\boxed{\frac{11}{60} \text{ cm}^2}$.
- At *92 minutes*, $h = \frac{92}{60}$. Therefore, the area of the puddle is $\boxed{\frac{1012}{60} \text{ cm}^2}$.
- At *5 hours*, $h = 5$. Therefore, the area of the puddle is $11 \cdot 5 = \boxed{55 \text{ cm}^2}$.
- At *1 day*, $h = 24$. Therefore, the area of the puddle is $11 \cdot 24 = \boxed{264 \text{ cm}^2}$.

The area of a circle is $A = \pi r^2$. Assuming the puddle is a perfect circle, we can find the radius as:

$$A = \pi r^2 \rightarrow \frac{A}{\pi} = r^2 \rightarrow \sqrt{\frac{A}{\pi}} = r \rightarrow r = \sqrt{\frac{A}{\pi}}$$

Our derived formula can be used to find the radius of each puddle.

- At *1 minute*, $A = \frac{11}{60}$. The radius is then $\boxed{r = \sqrt{\frac{11}{60\pi}} \approx 0.24 \text{ cm.}}$
- At *92 minutes*, $A = \frac{1012}{60}$. The radius is then $\boxed{r = \sqrt{\frac{1012}{60\pi}} \approx 2.32 \text{ cm.}}$

- At *5 hours*, $A = 5$. The radius is then $\boxed{\sqrt{\frac{55}{\pi}} \approx 4.18 \text{ cm.}}$

- At *1 day*, $A = 264$. The radius is then $\boxed{\sqrt{\frac{264}{\pi}} \approx 9.17 \text{ cm.}}$

1.12b Question: Is the radius of the puddle increasing at a constant rate?

1.12b Solution: Let us write the relationship between h, the number of hours, and r, the radius of the puddle. Using the derived formula above, this comes out to be:

$$r = \sqrt{\frac{11 \cdot h}{\pi}}$$

Fundamentally, this represents a square-root relationship $y = \sqrt{ax}$, not a linear one as the result of a constant rate ($y = ax$). Therefore, $\boxed{\text{the puddle is not increasing at a constant rate.}}$