Collingwood 37

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7.16 Context: Sven starts walking due south at 5 feet per second from a point 120 feet north of an intersection. At the same time Rudyard starts walking due east at 4 feet per second from a point 150 feet west of the intersection.

7.16a Problem: Write an expression for the distance between Sven and Rudyard t seconds after they start walking.

7.16a Solution: Let $(0,0)$ be the location of the intersection. Therefore, Sven begins at $(0,120)$ and Rudyard begins at $(-150,0)$. Since Sven moves down in our coordinate system at 5 feet per second, his location can be represented by $(0,120-5t)$, where t is the number of seconds that has elapsed. Similarly, since Rudyard moves right in our coordinate system at 4 feet per second, his location can be represented by $(-150+4t,0)$. The distance between these two points is then:

$$\begin{aligned}&\sqrt{(150-4t)^2+(120-5t)^2}\\&=\sqrt{22500-1200t+16t^2+14400-1200t+25t^2}\\&=\sqrt{41t^2-2400t+36900}\end{aligned}$$

Therefore, the expression for the distance between Sven and Rudyard is $\boxed{\sqrt{41t^2-2400t+36900}\text{ ft.}}$.

7.16b Problem: When are Sven and Rudyard closest? What is the minimum distance between them?

7.16b Solution: The minimum value of $\sqrt{41t^2-2400t+36900}$ is the minimum value of $41t^2-2400t+36900$ (for non-negative values of $41t^2-2400t+36900$). The vertex of $41t^2-2400t+36900$ is $-\frac{-2400}{2(41)}=\frac{1200}{41}\approx 29.26829$. Plugging this into the original expression is $\sqrt{41(29.26829)^2-2400(29.26829)+36900}\approx 42.16692$. Therefore, Sven and Rudyard are the closest at $\boxed{\approx 29.26829\text{ seconds}}$, at which they are $\boxed{\approx 42.16692\text{ feet apart}}$.

7.17 Problem: After a vigorous soccer match, Tina and Michael decide to have a glass of their favorite refreshment. They each run in a straight line along the indicated paths at a speed of 10 ft/sec. Parametrize the motion of Tina and Michael individually. Find when and where Tina and Michael are closest to one another; also compute this minimum distance.

7.17 Solution: Michael begins at (0,0) and heads to the soy milk at (200, 300). The slope in that case is $\frac{3}{2}$. We can find his horizontal and vertical movement in one second, $(3x)^2+(2x)^2=10^2\rightarrow 4x^2+9x^2=100\rightarrow x^2=\frac{100}{13}\rightarrow x=\sqrt{\frac{100}{13}}\approx 2.773501$. Therefore, he travel $2\cdot 2.773501=5.547002$ feet horizontally vertically and $3\cdot 2.773501=8.320503$. Therefore, the parametric equation representing his location is $\boxed{(5.547t, 8.321t)}$.

Tina begins at (400, 50) and goes to the beet juice, located at $(-50,275)$. The slope is $\frac{275-50}{-50-400}=-\frac{1}{2}$. This means that she travels two units left for every unit up she travels; therefore, we can write that $x^2+(2x)^2=10^2$ to find the vertical and horizontal components of her movement every second, in which she moves 10 feet in that direction, where x is the length of her vertical movement. This yields $5x^2=100\rightarrow x=\pm\sqrt{20}$; because the length must be positive, we know that Tina travels $2\sqrt{20}\approx 8.944272$ feet left and $\sqrt{20}\approx 4.472136$ feet up every second. Therefore, Tina's position can be represented by taking into account her initial position, yielding $\boxed{(400 - 8.944t, 50 + 4.472t)}$.

The distance between the two can be represented as:

$$\begin{aligned} &\sqrt{(400 - 8.944t - 5.547t)^2 + (50 + 4.472t - 8.321t)^2} \\ =&\sqrt{(-14.491t + 400)^2 + (-3.849t + 50)^2} \\ =&\sqrt{209.989081t^2 - 11592.8t + 160000 + 14.814801t^2 - 384.9t + 2500} \\ =&\sqrt{224.803882t^2 - 11977.7t + 162500} \end{aligned}$$

The vertex of $\sqrt{f(x)}$ is the vertex of $f(x)$, as long as the vertex of $f(x)$ is non-negative. Hence, we can find the vertex of $224.803882t^2 - 11977.7t + 162500$ using the vertex formula:

$$t = -\frac{-11977.7}{2(224.803882)} \approx 26.640332$$

Plugging this time back into the equation for their distance:

$$\sqrt{224.803882(26.640332)^2 - 11977.7(26.640332) + 162500} \approx 54.360363$$

Therefore, they are the closest at about $\boxed{\text{26.64 seconds and 54.36 feet apart}}$.