Collingwood 35

Andre Ye

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7.12 Problem: Jun has 300 meters of fencing to make a rectangular enclosure. She also wants to use some fencing to split the enclosure into two parts with a fence parallel to two of the sides. What dimensions should the enclosure have to have the maximum possible area?

7.12 Solution: Let the rectangular enclosure has length l and width w. The perimeter is 300 meters; thus it must be true that $3l+2w=300 \rightarrow w=150-\frac{3}{2}l$. The area of the enclosure is $l \cdot w$; plugging in our derived expression for l yields $\left(150-\frac{3}{2}l\right) \cdot l = -\frac{3}{2}l^2+150l$. The maximum value of this, using the vertex formula, occurs when $l=-\frac{150}{2\left(-\frac{3}{2}\right)}=50$. Plugging this into the origin equation, we find that $w=150-\frac{3}{2} \cdot 50 = 75$. Therefore, the dimension of the enclosure should have a maximum possible area of $\boxed{\text{75 meters by 50 meters.}}$

7.13 Problem: You have \$6000 with which to build a rectangular enclosure with fencing. The fencing material costs \$20 per meter. You also want to have two partitions across the width of the enclosure, so that there will be three separated spaces in the enclosure. The material for the partitions costs \$15 per meter. What is the maximum area you can achieve for the enclosure?

7.13 Solution: Let the length and width of the rectangular enclosure be l and w; thus, there are two partitions of length w along the rectangle. The total cost is $20(2l+2w)+15(2w)$, since the outside perimeter (which is $2l+2w$ meters) costs 20 dollars per meter and the partition (which is $2w$ meters in total) costs 15 dollars per meter. The following equation must be true: $20(2l+2w)+15(2w)=6000$. Expanding yields $40l+40w+30w=6000 \rightarrow 40l+70w=6000 \rightarrow 4l+7w=600 \rightarrow l=150-\frac{7}{4}w$. The area of the enclosure is $l \cdot w$, which yields $\left(150-\frac{7}{4}w\right)w=-\frac{7}{4}w^2+150w$. The maximum value of w, using the vertex formula, occurs at $-\frac{150}{2\left(-\frac{7}{4}\right)}=\frac{300}{7}$. This yields a length of $l=1500-\frac{7}{4}\left(\frac{300}{7}\right)=75$. Multiplying these yields an area of about $\boxed{\text{3214.2857 meters squared}}$.