Collingwood 33

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7.7 Context: A hot air balloon takes off from the edge of a plateau. Impose a coordinate system as pictured below and assume that the path the balloon follows is the graph of the quadratic function $y = f(x) = -\frac{4}{2500}x^2 + \frac{4}{5}x$. The land drops at a constant incline from the plateau at the rate of 1 vertical foot for each 5 horizontal feet.

7.7a Problem: What is the maximum height of the balloon above plateau level?

7.7a Solution: The vertex if $-\frac{b}{2a} = -\frac{\frac{4}{5}}{2\left(-\frac{4}{2500}\right)} = 250$. This yields a height of $-\frac{4}{2500}\left(250\right)^2 + \frac{4}{5}\left(250\right) = 100$. Therefore the maximum height of the balloon above plateau level is $\boxed{100 \text{ feet}}$.

7.7b Problem: What is the maximum height of the balloon above ground level?

7.7b Solution: The plateau can be represented with the line $y = -\frac{1}{5}x$. Therefore, the total height is $-\frac{4}{2500}x^2 + \frac{4}{5}x + \left(\frac{1}{5}x\right) = -\frac{1}{625}x^2 + x$. The maximum is at $x = -\frac{b}{2a} = -\frac{1}{2\left(-\frac{1}{625}\right)} = \frac{625}{2}$. Plugging this yields $-\frac{1}{625}\left(\frac{625}{2}\right)^2 + \frac{625}{2} = -\frac{625}{4} + \frac{625}{2} = \frac{625}{4}$. Therefore, the maximum height is at $\boxed{\frac{625}{4} \text{ feet}}$.

7.7c Problem: Where does the balloon land on the ground?

7.7c Solution: The solution representing the distance from the balloon to the ground was $-\frac{1}{625}x^2 + x$, as derived in 7.7b solution. The balloon lands on the ground when the distance is 0. Solving:

$$\begin{aligned} -\frac{1}{625}x^2 + x &= 0 \\ x^2 - 625x &= 0 \\ x &= \frac{625 \pm \sqrt{625^2}}{2} \\ &= 0, 625 \end{aligned}$$

$x = 0$ is when the balloon initially lands on the ground. Therefore, the solution for when the balloon lands on the ground afterwards is at $x = 625$. Hence, y is $-\frac{1}{5}(625) = -125$. The point where it lands is now $\boxed{(625, -125).}$

7.7d Problem: Where is the balloon 50 feet above the ground?

7.7d Solution: Solving for the following equation:

$$\begin{aligned} -\frac{1}{625}x^2 + x &= 50 \\ x^2 - 625x &= -31250\cdot \\ x^2 - 625x + 31250 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-(-625) \pm \sqrt{(-625)^2 - 4\cdot\ 1\cdot\ 31250}}{2\cdot\ 1} \\ &= \frac{625 + 125\sqrt{17}}{2}, \frac{625 - 125\sqrt{17}}{2} \end{aligned}$$

These are 570.19 and 54.81 when rounded, respectively. The location of each is $-\frac{4}{2500}(570.19)^2 + \frac{4}{5}(570.19) \approx -64.04$ and $-\frac{4}{2500}(54.81)^2 + \frac{4}{5}(54.81) \approx 39.04$. Therefore, the two locations where the balloon is 50 feet above the ground are $\boxed{(570.19, -64.04) \text{ and } (54.81, 39.04)}$.

7.8a Problem: Suppose $f(x) = 3x^2 - 2$. Does the point (1,2) lie on the graph of $y = f(x)$? Why or why not?

7.8a Solution: $3(1)^2 - 2 = 1$. Because $1 \neq 2$, the point (1, 2) $\boxed{\text{does not}}$ lie on the graph.

7.8b Problem: If b is a constant, where does the line $y = 1 + 2b$ intersect the graph of $y = x^2 + bx + b$?

7.8b Solution: Setting the two equal to each other:

$$\begin{aligned} x^2 + bx + b &= 1 + 2b \\ x^2 + bx - b - 1 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4(1)(-b-1)}}{2} \\ &= \frac{-b \pm \sqrt{b^2 + 4b + 4}}{2} \\ &= \frac{-b \pm (b+2)}{2} \end{aligned}$$

The two, when simplified, are $\frac{-b+(b+2)}{2} = 1$ and $\frac{-b-(b+2)}{2} = -b - 1$. These yield y-values $1 + 2b$; therefore the two intersections are $\boxed{(1, 1 + 2b \text{ and } (-b - 1, 1 + 2b)}$.

7.8c Problem: If a is a constant, where does the line $y = 1 - a^2$ intersect the graph of $y = x^2 - 2ax + 1$?

7.8c Solution: Setting these two equal to each other:

$$\begin{aligned} x^2 - 2ax + 1 &= 1 - a^2 \\ x^2 - 2ax + a^2 &= 0 \\ (x - a)^2 &= 0 \\ x &= a \end{aligned}$$

Therefore, the point of intersection is $\boxed{(a, 1 - a^2)}$ (the y-value determined by $y = 1 - a^2$).

7.8d Problem: Where does the graph of $y = -2x^2 + 3x + 10$ intersect the graph of $y = x^2 + x - 10$?

7.8d Solution: Setting these two equal to each other;

$$\begin{aligned}
x^2 + x - 10 &= -2x^2 + 3x + 10 \\
3x^2 - 2x - 20 &= 0 \\
x &= \frac{-(-2) \pm\ 2\sqrt{61}}{2 \cdot 3} \\
&= \frac{1+\sqrt{61}}{3}, \frac{1-\sqrt{61}}{3}
\end{aligned}$$

These, when rounded, yield -2.2701 and 2.9368. These yield values of $(-2.2701)^2+(-2.2701)-10 = -7.1167$ and $(2.9368)^2+(2.9368)-10 = 1.5612$. Therefore, the solutions are $\boxed{(-2.2701, -7.1168) \text{ and } (2.9368, 1.5612)}$.