Collingwood 32

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7.5 Context: The initial price of buzz.com stock is 10 dollars per share. After 20 days the stock price is 20 dollars per share and after 40 days the price is 25 dollars per share. Assume that while the price of the stock is not zero it can be modeled by a quadratic function.

7.5a Problem: Find the multipart function $s(t)$ giving the stock price after t days. If you buy 1000 shares after 30 days, what is the cost?

7.5a Solution: We can use these three points $(0, 10)$, $(20, 20)$, and $(40, 25)$. A quadratic must take the form $y = ax^2 + bx + c$; therefore we can write the following linear equations:

$$\begin{cases} 0a + 0b + c = 10 \to c = 10 \\ 400a + 20b + c = 20 \\ 1600a + 40b + c = 25 \end{cases}$$

Because $c = 10$, we can write the other two equations as $400a + 20b + 10 = 20$ and $1600a + 40b + 10 = 25$; these can be simplified as $400a + 20b = 10 \to 40a + 2b = 1$ and $1600a + 40b = 15 \to 320a + 8b = 3$. Using elimination, the first equation can be rewritten as $160a + 8b = 4$; subtracting this from the second equation yields $160a = -1 \to a = -\frac{1}{160}$. Plugging this into $40a + 2b = 1$ yields $40\left(-\frac{1}{160}\right) + 2b = 1 \to -\frac{1}{4} + 2b = 1 \to 2b = \frac{5}{4} \to b = \frac{5}{8}$. Therefore, the equation modelling the stock over time is $\boxed{s(x) = -\frac{1}{160}x^2 + \frac{5}{8}x + 10}$. However, this parabola points down (coefficient of x^2 is negative), the domain cannot include values of x for which the corresponding value is negative. We know that the lower bound of the domain is 0, since the value of the stock does not exist for days prior to the initial release of the stock. To find the upper bound, we need to find its roots.

$$\begin{aligned} -\frac{1}{160}x^2 + \frac{5}{8}x + 10 &= 0 \\ -x^2 + 100x + 1600 &= 0 \\ x^2 - 100x - 1600 &= 0 \\ x^2 - 100x &= 1600 \\ x^2 - 100x + 2500 &= 1600 + 2500 \\ (x-50)^2 &= 4100 \\ x - 50 &= \pm 10\sqrt{41}x & &= 50 \pm 10\sqrt{41} \end{aligned}$$

We are looking for the larger root, which is $50 + 10\sqrt{41}$. Therefore, we can construct our multi-part function is:

$$s(t) = \begin{cases} -\frac{1}{160}x^2 + \frac{5}{8}x + 10 & \text{if } 0 \le x \le 50 + 10\sqrt{41} \\ 0 & \text{if } x > 50 + 10\sqrt{41} \end{cases}$$

Therefore, at 30 days, the price of a stock is $-\frac{1}{160}(900) + \frac{5}{8}30 + 10 = \frac{180}{8} = 23.125$. 100 shares of this yields $\boxed{\$23,125 \text{ price}}$ to purchase 100 shares of the stock after 30 days.

7.5b Problem: To maximize profit, when should you sell shares? How much will the profit be on your 1000 shares purchased in (a)?

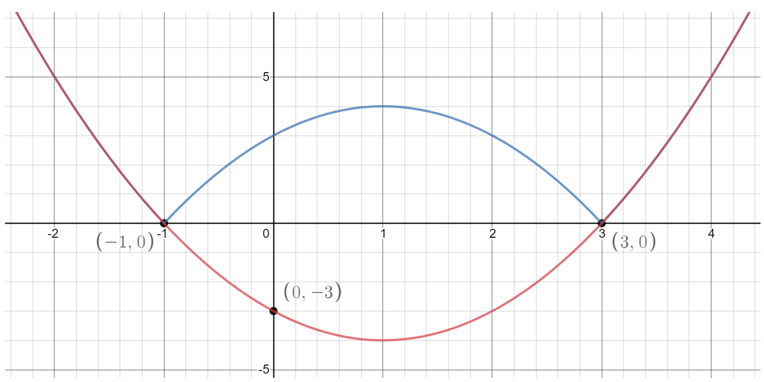
7.5b Solution: The maximum profit is the vertex of the quadratic $y = -\frac{1}{160}x^2 + \frac{5}{8}x + 10$. Since we personally would not like to deal with gnarly fractions, let us solve a slightly more convenient linear equation for the derivative:

$$\begin{aligned} 0 &= -\frac{2}{160}x + \frac{5}{8} \\ \frac{1}{80}x &= \frac{5}{8} \\ x &= \frac{5}{8} \cdot 80 = 50 \end{aligned}$$

Therefore, at $\boxed{\text{50 days}}$, the stock has its maximum value $-\frac{1}{160}2500 + \frac{5}{8}50 + 10 = 25.625$; with 1000 shares this has a value of 25,625 dollars. The difference between the starting and selling value yields a profit of $\boxed{\text{2,500 dollars.}}$

7.7 Problem: Sketch the graph of $y = x^2 - 2x - 3$. Label the coordinates of the x and y intercepts of the graph. In the same coordinate system, sketch the graph of $y = |x^2 - 2x - 3|$, give the multipart rule and label the x and y intercepts of the graph.

7.7 Solution: The x-intercepts are the roots, which can be derived by $0 = x^2 - 2x - 3 = (x-3)(x+1) \rightarrow x = -1, 3$; hence they are $(-1, 0)$ and $(3, 0)$. The y-intercept is $(0, -3)$. Below: red represents $y = x^2 - 2x - 3$, blue represents $y = |x^2 - 2x - 3|$.



The multipart function of $|x^2 - 2x - 3|$ consists of several parts such that y is never less than 0:

- On the left of the smaller root the function is normal. Rephrased in math language: $y = x^2 - 2x - 3$ for $x < -1$.
- On the right of the larger root the function is normal. Rephrased in math language: $y = x^2 - 2x - 3$ for $x > 3$.
- Between the small and large roots, the function is the reflection of the 'normal' function over the x axis. This is $-\left(x^2 - 2x - 3\right)^2 = -x^2 + 2x + 3$. Therefore, the multi-part function is $-x^2 + 2x + 3$ from $-1 \leq g \leq 3$.

Combining these, the multipart function is:

$$|x^2 - 2x - 3| = \begin{cases} x^2 - 2x - 3 & \text{if } x < -1 \\ -x^2 + 2x + 3 & \text{if } -1 \leq x \leq 3 \\ x^2 - 2x - 3 & \text{if } x > 3 \end{cases}$$