Collingwood 31

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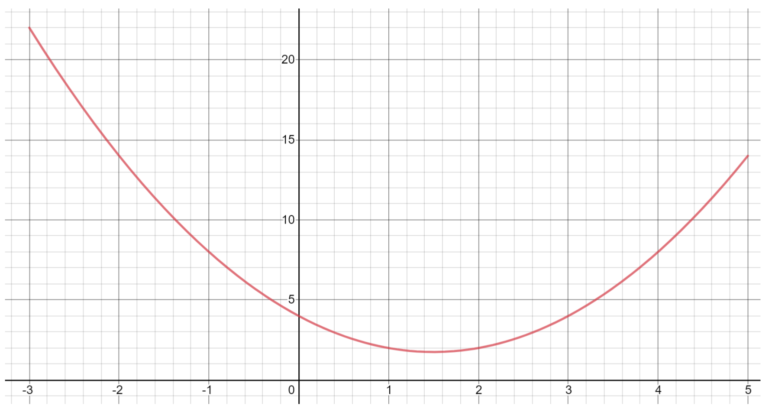
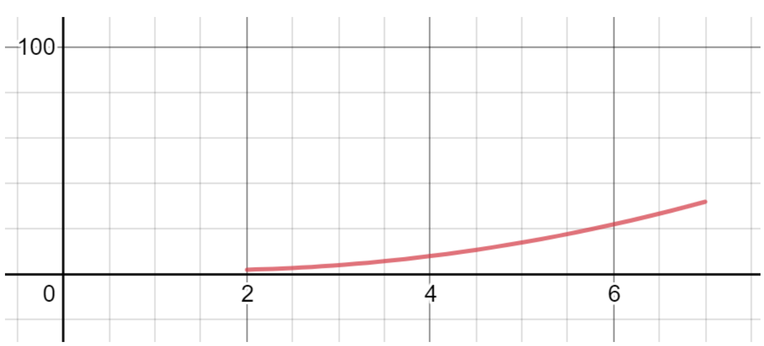
7.3a Problem: Sketch the graph of the function $f(x) = x^2 - 3x + 4$ on the interval $-3 \leq x \leq 5$. What is the maximum value of $f(x)$ on that interval? What is the minimum value of $f(x)$ on that interval?

7.3a Solution: The maximum value of x is at $x = -3$. This evaluates to $9 + 9 + 4 = 22$; therefore the maximum value of $f(x)$ is 22. The minimum value, on the other hand, is the vertex, which is $x = -\frac{b}{2a} = -\frac{-3}{2} = \frac{3}{2}$. This is $\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 4 = \frac{7}{4}$. Therefore, the $\boxed{\text{max. is 22 and the min. is } \frac{7}{4}}$.



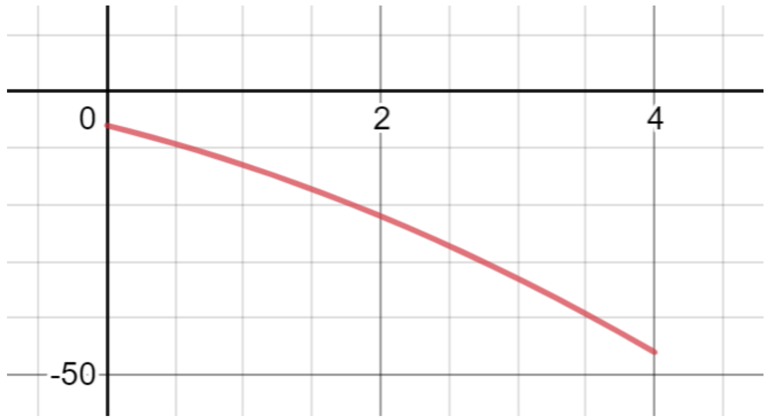
7.3b Problem: Sketch the graph of the function $f(x) = x^2 - 3x + 4$ on the interval $2 \leq x \leq 7$. What is the maximum value of $f(x)$ on that interval? What is the minimum value of $f(x)$ on that interval?

7.3b Solution: The maximum is at $x = 7$; this value is $49 - 21 + 4 = 32$. The minimum is at the vertex $x = -\frac{b}{2a} = -\frac{-3}{2} = \frac{3}{2}$; this is less than the lower bound of the domain, $x = 2$. Therefore the minimum is $x = 2$ with value $4 - 6 + 4 = -6$. Hence, the $\boxed{\text{max. is 32 and the min. is } -6}$.



7.3c Problem: Sketch the graph of the function $g(x) = -(x + 3)^2 + 3$ on the interval $0 \leq x \leq 4$. What is the maximum value of $g(x)$ on that interval? What is the minimum value of $g(x)$ on that interval?

7.3c Solution: The maximum is the vertex, at $(-3, 3)$; this is at a value of x less than the smallest number in the bound. Therefore, the maximum occurs at $x = 0$ (the lower bound), which has value $-9 + 3 = -6$. The minimum value is at the upper bound $x = 4$, with value $-49 + 3 = -46$. Hence, the $\boxed{\text{max. is } -6 \text{ and the min. is } -46}$.



7.4 Problem: If the graph of the quadratic function $f(x) = x^2 + dx + 3d$ has its vertex on the x-axis, what are the possible values of d? What if $f(x) = x^2 + 3dx - d^2 + 1$?

7.4 Solution: If the vertex of $x^2 + dx + 3d$ is on the x axis, then if written in the form $a(x - h)^2 + k$, then $k = 0$. Therefore, $x^2 + dx + 3d$ can be factored without any remainder. We know that $a = 1$ because the coefficient of the quadratic in standard form for x^2 is 1; therefore the vertex form is $(x - h)^2$, which can be expanded as $x^2 - 2hx + h^2$. Aligning this with $x^2 + dx + 3d$, yields $-2h = d$ and $h^2 = 3d$. Combining these two yields $h^2 = 3(-2h) \to h^2 = -6h \to h = 0, -6$. Therefore:

- When $h = 0$, $(x - h)^2 = x^2$. It is clear that $d = 0$.
- When $h = -6$, $(x - h)^2 = x^2 + 12x + 36$. It is clear that $d = 12$.

Therefore, $\boxed{d = 0 \text{ and } 12}$.

Considering the latter proposed case, $f(x) = x^2 + 3dx - d^2 + 1$, we know from the expansion of $(x - h)^2$ (as discussed above) that $3d = -2h$ and $-d^2 + 1 = h^2$. Using substitution, $-d^2 + 1 = \left(-\frac{3}{2}d\right)^2 \to -d^2 + 1 = \frac{9}{4}d^2 \to \frac{4}{15} = d^2 \to d = \pm\frac{2}{\sqrt{15}}$. Therefore, in this case, $\boxed{d = \pm\frac{2}{\sqrt{15}}.}$