Collingwood Homework 3

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6 October 2020

1.8 Context: The famous theory of relativity predicts that a lot of weird things will happen when you approach the speed of light $c = 3 \times 10^8$ m/sec. For example, here is a formula that relates the mass m_0 (in kg) of an object at rest and its mass when it is moving at a speed v:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

1.8a Question: Suppose the object moving is Dave, who has a mass of $m_o = 66$ kg at rest. What is Dave's mass at 90% of the speed of light? At 99% the speed of light? At 99.9% the speed of light?

1.8a Solution: The problem compiles necessary variables for us.

- $m_0 = 66$ kg
- $c = 3 \times 10^8$
- $v = 0.9c$ (at 90% of the speed of light)

To find Dave's mass at other speeds of light (99% and 99.9%), v can be written as $0.99c$ and $0.999c$, respectively. The variables can subsequently be put into the given formula. Let m_n denote the mass of Dave when he moves at the nth percent of light.

$$m_{90} = \frac{66}{\sqrt{1 - \frac{(0.9\times3\times10^8)^2}{(3\times10^8)^2}}} \qquad m_{99} = \frac{66}{\sqrt{1 - \frac{(0.99\times3\times10^8)^2}{(3\times10^8)^2}}} \qquad m_{99.9} = \frac{66}{\sqrt{1 - \frac{(0.999\times3\times10^8)^2}{(3\times10^8)^2}}}$$

$$m_{90} = \frac{66}{\sqrt{1 - 0.81}} \qquad m_{99} = \frac{66}{\sqrt{1 - 0.9801}} \qquad m_{99.9} = \frac{66}{\sqrt{1 - 0.998}}$$

$$m_{90} = \frac{66}{\sqrt{0.19}} \qquad m_{99} = \frac{66}{\sqrt{0.0199}} \qquad m_{99.9} = \frac{66}{\sqrt{0.002}}$$

$$m_{90} \approx 151.41 \text{ kg} \qquad m_{99} \approx 467.86 \text{ kg} \qquad m_{99.9} \approx 1475.80 \text{ kg}$$

Thus, the mass of Dave when he travels at various speeds is:

- $\boxed{\approx 151.41 \text{ kg when he travels at 90\% the speed of light}}$
- $\boxed{\approx 467.86 \text{ kg when he travels at 99\% the speed of light}}$
- $\boxed{\approx 1475.80 \text{ kg when he travels at 99.9\% the speed of light}}$

1.8b Question: How fast should Dave be moving to have a mass of 500 kg?

1.8b Solution: Dave would like to have a mass of 500 kg ($m = 500$ kg) when travelling at speed v. The following equation models this problem:

$$500 = \frac{66}{\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}}$$

Moving the bottom half of the right side of the equation to make finding the value of v easier:

$$\sqrt{1-\frac{v^2}{\left(3\times 10^8\right)^2}}=\frac{66}{500}=\frac{33}{250}$$

Then, making algebraic manipulations and simplifications to solve for v:

$$1-\frac{v^2}{\left(3\times 10^8\right)^2}=\frac{1089}{62500}$$

$$-\frac{v^2}{\left(3\times 10^8\right)^2}=\frac{1089}{62500}-1=\frac{1089}{62500}-\frac{62500}{62500}=-\frac{61411}{62500}$$

$$v^2=\frac{61411}{62500}\left(3\times 10^8\right)^2$$

$$v=\sqrt{\frac{61411}{62500}}\left(3\times 10^8\right)$$

$$v=0.9912\left(3\times 10^8\right)\approx 2.97\times 10^8$$

Therefore, if Dave would like to have a mass of 500 kg, he needs to be moving at about $\boxed{2.97\times 10^8 \text{ m/sec.}}$

1.10 Question: Aleko's Pizza has delivered a beautiful 16 inch diameter pie to Lee's dorm room. The pie is sliced into 8 equal sized pieces, but Lee is such a non-conformist he cuts off an edge as pictured. John then takes one of the remaining triangular slices. Who has more pizza and by how much?

1.10 Solution: Ignoring the faulty assumption that pizzas and pies are equivalent, let us first find the area of John's part. This will assist us in finding the area of Lee's part. John's part is an isosceles right triangle with the hypotenuse as the radius of the circle. We can verify that the triangle is isosceles because the non-right angle is $\frac{360^\circ \text{ degrees in a circle}}{8 \text{ pieces in the pie}}=45^\circ$. This means the other angle must be $180^\circ-45^\circ-90^\circ=45^\circ$ as well.

The hypotenuse of an isosceles right triangle can be written as $s\sqrt{2}$, where s is the side length. Because the hypotenuse is the radius of the circle, which is $\frac{16}{2}=8$ inches, the sides of the triangle must be $\frac{8}{\sqrt{2}}=4\sqrt{2}$ inches long.

Thus, the area of the triangle is $\frac{(4\sqrt{2})^2}{2}=16$ inches^2.

Considering only one half of Lee's part, we see that it is one sector (one-eighth) of the circle, minus John's area. The area of the circle is $8^2\pi=64\pi\text{in.}^2$, so the area of one sector is $\frac{64\pi}{8}=8\pi$. Subtracting John's part from this yields $8\pi-16$; accounting for both of Lee's parts, we derive $2(8\pi-16)=16\pi-32\approx 18.27$ inches^2.

Therefore, $\boxed{\text{Lee's slice is larger than John's by about 2.27 inches}^2.}$