Collingwood 27

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6.9 Problem: A baseball diamond is a square with sides of length 90 ft. Assume Edgar hits a home run and races around the bases (counterclockwise) at a speed of 18 ft/sec. Express the distance between Edgar and home plate as a function of time t. (Hint: This will be a multi-part function.) Try to sketch a graph of this function.

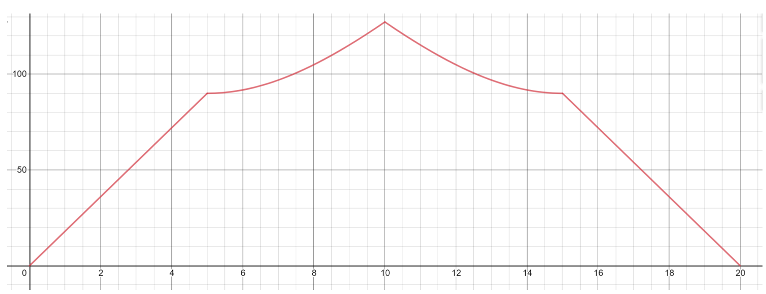
6.9 Solution: Let the home plate be located at $(0, 0)$, and let us orient the square baseball field such that first base is located at $(90, 0)$. Thus, we can construct equations representing Edgar's location at a time t:

- From $0 \leq t < 5$, Edgar runs horizontally along $y = 0$. Thus his position can be represented by $(18t, 0)$.
- From $5 \leq t < 10$, Edgar runs vertically along $x = 90$. This position can be represented by $(90, 18(t-5))$.
- From $10 \leq t < 15$, Edgar runs horizontally along $y = 90$. Thus his position can be represented by $(90 - 18(t - 10), 90)$.
- From $15 \leq t \leq 20$, Edgar runs vertically along $x = 0$. This position can be represented by $(0, 90 - 18(t - 15))$.

Thus, using the distance formula, the multipart function $d(t)$ representing Edgar's distance at time t (in seconds) from the home plate can be constructed as:

$$d(t) = \begin{cases} 18t & \text{if } 0 \leq t < 5 \\ \sqrt{90^2 + (18(t-5))^2} & \text{if } 5 \leq t < 10 \\ \sqrt{(90 - 18(t-10))^2 + 90^2} & \text{if } 10 \leq t < 15 \\ 90 - 18(t - 15) & \text{if } 15 \leq t \leq 20 \end{cases} \tag{1}$$

The graph of this function is:



6.10 Context: Pagliacci Pizza has designed a cardboard delivery box from a single piece of cardboard, as pictured.

6.10a Problem: Find a polynomial function $v(x)$ that computes the volume of the box in terms of x. What is the degree of v?

6.10a Solution: The box has three attributes: the length, the width, and the height, which determine its volume.

- *Length* of the box can be found by considering half of it, which has length 25. Subtracting one square yields a length of $25 - x$.
- *Width* of the box can be found by $20 - 2x$, as demonstrated in the diagram.
- *Height* of the box is simply x.

Multiplying these together yields $(25-x)\cdot(20-2x)\cdot(x) = x\left(2x^2 - 70x + 500\right) = 2x^3 - 70x^2 + 500x$. Therefore, the polynomial representing the volume of the box in terms of x is $\boxed{v(x) = 2x^3 - 70x^2 + 500x \text{ with degree 3.}}$

6.10b Problem: Find a polynomial function $a(x)$ that computes the exposed surface area of the closed box in terms of x. What is the degree of a? What are the explicit dimensions if the exposed surface area of the closed box is 600 sq. inches?

6.10b Solution: To find the exposed surface area, we need to add a few components together.

- There are four horizontal rectangles depicted in the diagram, but only two are exposed. These each of the width $25 - x$ and height x; thus their area is $(25 - x) \cdot x \cdot 4 = 2\left(-x^2 + 25x\right) = -2x^2 + 50x$.
- There are two vertical rectangles in the diagram. These each of width x and height $20 - 2x$; thus their area is $x \cdot (20 - 2x) \cdot 2 = 2\left(-2x^2 + 20x\right) = -4x^2 + 40x$.
- There are two large rectangles in the diagram. These each have width $25 - x$ and height $20 - 2x$; thus their area is $(25 - x) \cdot (20 - 2x) \cdot 2 = 2\left(2x^2 - 70x + 500\right) = 4x^2 - 140x + 1000$.

Adding these three elements together yields a polynomial solution: $\left(-2x^2 + 50x\right) + \left(-4x^2 + 40x\right) + \left(4x^2 - 140x + 1000\right) = -2x^2 - 50x + 1000$. Thus, the polynomial is $\boxed{a(x) = -2x^2 - 50x + 1000 \text{ with degree 2}}$. To find when the exposed surface area is equal to 600 square inches, we can write and solve the following equation:

$$
\begin{aligned}
-2x^2 - 50x + 1000 &= 600 \\
-2x^2 - 50x + 400 &= 0 \\
2x^2 + 50x - 400 &= 0 \\
x^2 + 25x - 200 &= 0 \\
x &= \frac{-25 \pm \sqrt{25^2 - 4(1)(-200)}}{2(1)} \\
&= \frac{-25 \pm 5\sqrt{57}}{2}
\end{aligned}
$$

The height (which is x) cannot be negative, so the solution is $\frac{-25+5\sqrt{57}}{2} \approx 6.374586$. Therefore, we can calculate the dimensions of the box to be:

- Height $= x \approx 6.374586$ *inches*
- Length $= 25 - x \approx 18.625414$ *inches*
- Width $= 20 - 2x \approx 7.250828$ *inches*

Thus, the dimensions of the box are $\boxed{6.374586 \times 18.625414 \times 7.250828 \text{ inches.}}$