Collingwood 25

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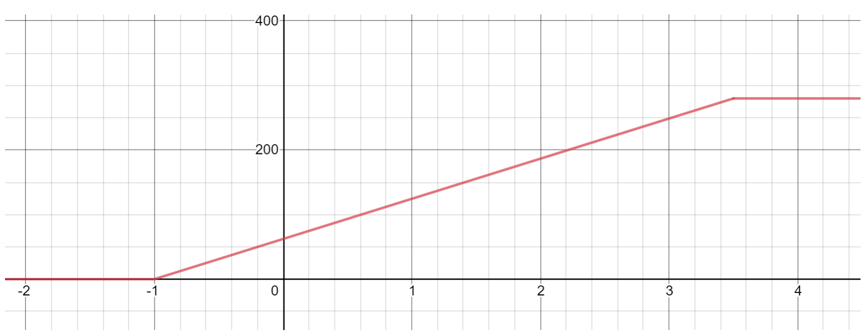
6.7 Context: This problem deals with cars traveling between Bellevue and Spokane, which are 280 miles apart. Let t be the time in hours, measured from 12:00 noon; for example, $t = -1$ is 11:00 am.

6.7a Problem: Joan drives from Bellevue to Spokane at a constant speed, departing from Bellevue at 11:00 am and arriving in Spokane at 3:30 pm. Find a function $j(t)$ that computes her distance from Bellevue at time t. Sketch the graph, specify the domain and determine the range.

6.7a Solution: If Joan begins in Bellevue, she begins 0 miles away from it at time $t = -1$. She travels to Spokane, 280 miles apart, and reaches there at 3:30 pm, or $t = 3.5$. Thus, two points can be used to indicate her position: $(-1, 0)$ and $(3.5, 280)$. Because she travels at a constant rate, we can express the distance as a line, $\frac{280}{3.5+1}(t + 1) = 62.\overline{2}(t + 1)$. However, before $t = -1$ and after $t = 3.5$, she remains at 0 and 260 miles away from Bellevue, respectively. The function is then:

$$j(t) = \begin{cases} 0 & \text{if } t \leq -1 \\ 62.\overline{2}(t+1) & \text{if } -1 < t < 3.5 \\ 280 & \text{if } t \geq 3.5 \end{cases} \tag{1}$$

The domain is all all real numbers, assuming for the purpose of this problem that Joan remains in Bellevue for all the time before she begins travelling to Spokane and remains in Spokane after she reaches there. The range is $[0, 280]$.

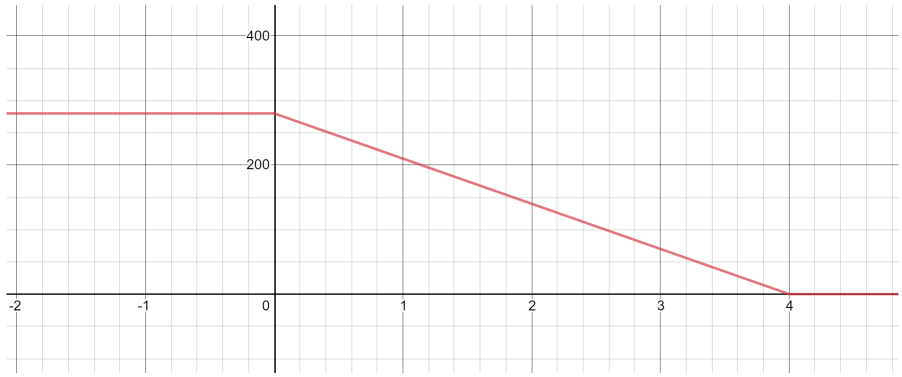


6.7b Problem: Steve drives from Spokane to Bellevue at 70 mph, departing from Spokane at 12:00 noon. Find a function $s(t)$ for his distance from Bellevue at time t. Sketch the graph, specify the domain and determine the range.

6.7b Solution: Steve begins at Spokane, meaning that he begins 280 miles from Bellevue at $t = 0$. He then drives to Bellevue at 70mph, meaning that the slope of a line representing his distance to Bellevue would be -70. Because he travels at a constant rate, the line representing his distance is $-70t + 280$. However, before $t = 0$ and after $t = 4$ (the point where $-70t + 280 = 0$ and he reaches Bellevue), he remains at $s(t) = 280$ and $s(t) = 0$, respectively. The function is then:

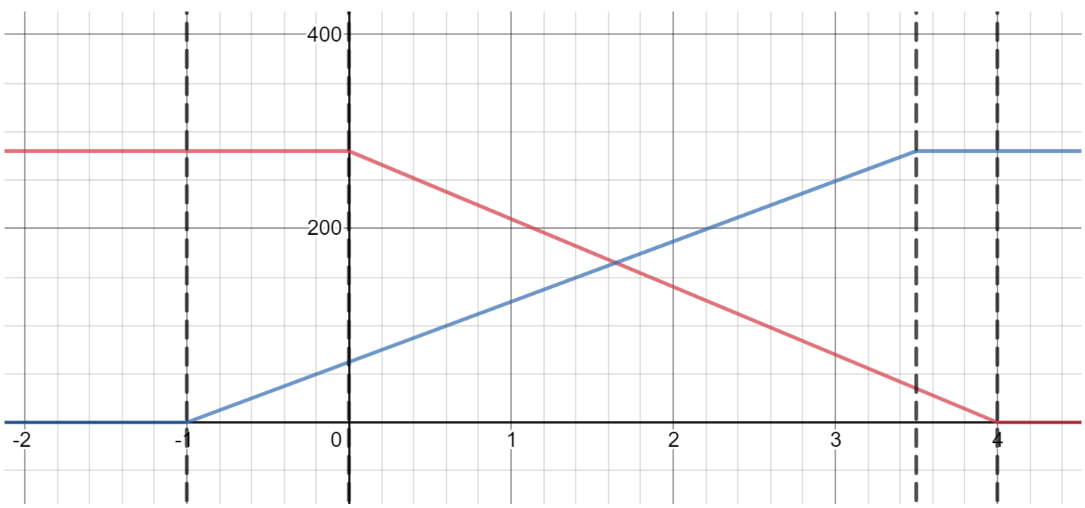
$$s(t) = \begin{cases} 280 & \text{if } t \leq 0 \\ -70t + 280 & \text{if } 0 < t < 4 \\ 0 & \text{if } t \geq 4 \end{cases} \tag{2}$$

The domain is all all real numbers, assuming for the purpose of this problem that Steve remains in Spokane for all the time before he begins travelling to Bellevue and remains in Bellevue after she reaches there. The range is $[0, 280]$.



6.7c Problem: Find a function $d(t)$ that computes the distance between Joan and Steve at time t.

6.7c Solution: There are a few possibilities, as outlined in the image below. We can partition the graph containing both lines into several sections, split by when a major change occurs in one of the lines. Every region between two of the dotted black lines (or to the left or right of the leftmost or rightmost one) is a section that needs to be accounted for.



- In the leftmost section, marked by $t \leq -1$, the distance between Steve and Joan remains constant, 280 miles apart.
- In the second section, marked by $-1 < t \leq 0$, Steve remains 280 miles from Bellevue, whereas Joan begins travelling $62.\overline{2}(t+1)$ miles closer to Spokane. The difference between the two is represented by $280 - 62.\overline{2}(t+1)$.
- In the third section, marked by $0 < t \leq 3.5$, Steve begins travelling towards Bellevue, whereas Joan continues travelling towards Spokane. Their difference can be represented by $|(62.\overline{2}(t+1)) - (-70t + 280)|$. This simplifies to $|\frac{1190}{9}t - \frac{1960}{9}|$.

- In the fourth section, marked by $3.5 < t \leq 4$, Joan has reached Spokane and is at a constant 280 miles away from Bellevue. Steve continues moving towards Bellevue, so their difference is $280 - (-70t + 280)$.
- In the fifth section, marked by $t > 4$, both Steve and Joan have reached their destinations. The difference between them remains 280 miles.

Assembling our findings, the multi-part function $d(t)$ that represents the difference between Joan and Steve at time t is:

$$d(t) = \begin{cases} 280 & \text{if } t \leq -1 \\ 280 - 62.\overline{2}(t+1) & \text{if } -1 < t \leq 0 \\ \left|\frac{1190}{9}t - \frac{1960}{9}\right| & \text{if } 0 < t \leq 3.5 \\ 70t & \text{if } 3.5 < t \leq 4 \\ 280 & \text{if } t > 4 \end{cases} \tag{3}$$

6.8 Problem: Arthur is going for a run. From his starting point, he runs due east at 10 feet per second for 250 feet. He then turns, and runs north at 12 feet per second for 400 feet. He then turns, and runs west at 9 feet per second for 90 feet. Express the (straight-line) distance from Arthur to his starting point as a function of t, the number of seconds since he started.

6.8 Solution: First, let us represent Arthur's position in terms of t. He begins at the point $(0,0)$ on our imposed coordinate system. During $0 \leq t < 25$, his position can be represented as $(10t, 0)$. He stops after $\frac{250}{10} = 25$ seconds. Then, from $25 \leq t < \frac{175}{3}$, his position can be represented as $(250, 12(t-25))$, ending at the point $(250, 400)$ after $25 + \frac{400}{12} = \frac{175}{3}$ total seconds. Lastly, he runs west from $\frac{175}{3} \leq t \leq \frac{205}{3}$ at 9 feet per second, and his position can be represented as $\left(250 - 9\left(t - \frac{175}{3}\right), 400\right)$. Using the distance formula, the multi-part function $d(t)$ that returns the distance Arthur is from his starting point at second t is:

$$d(t) = \begin{cases} 10t & \text{if } 0 \leq t < 25 \\ \sqrt{250^2 + (12(t-25))^2} & \text{if } 25 \leq t < \frac{175}{3} \\ \sqrt{\left(250 - 9\left(t - \frac{175}{3}\right)\right)^2 + 400^2} & \text{if } \frac{175}{3} \leq t \leq \frac{205}{3} \end{cases} \tag{4}$$