Collingwood 24

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6.3 Context: Solve each of the following equations for x.

6.3a Problem: $g(x) = 17$, where $g(x) = |3x + 5|$.

6.3a Solution: There are two equations that need to be solved for:

$$\begin{aligned} 17 &= 3x + 5 \\ 12 &= 3x \\ x &= 4 \end{aligned}$$

One acceptable value of x is 4.

$$\begin{aligned} -17 &= 3x + 5 \\ -22 &= 3x \\ -\frac{22}{3} &= x \end{aligned}$$

Hence, the two solutions are $\boxed{x = 4 \text{ and } x = -\frac{22}{3}.}$

6.3b Problem: $f(x) = 1.5$ where

$$f(x) = \begin{cases} 2x & \text{if } x < 3 \\ 4 - x & \text{if } x \geq 3 \end{cases} \tag{1}$$

6.3b Solution: There are two equations that need to be solved for:

$$\begin{aligned} \frac{3}{2} &= 2x \\ \frac{3}{4} &= x \end{aligned}$$

This satisfies the inequality $x < 3$.

$$\begin{aligned} \frac{3}{2} &= 4 - x \\ x &= 4 - \frac{3}{2} \\ x &= \frac{5}{2} \end{aligned}$$

This result does not satisfy the inequality $x \geq 3$. Therefore, the one solution is $\boxed{x = \frac{3}{4}}$.

6.3c Problem: $h(x) = -1$ where

$$f(x) = \begin{cases} -8 - 4x & \text{if } x \leq -2 \\ 1 + \frac{1}{3}x & \text{if } x > -2 \end{cases} \tag{2}$$

6.3b Solution: There are two equations that need to be solved for:

$$\begin{aligned} -8 - 4x &= -1 \\ -4x &= 7 \\ x &= -\frac{7}{4} \end{aligned}$$

This does not satisfy the inequality $x \leq -2$.

$$\begin{aligned} 1 + \frac{1}{3}x &= -1 \\ \frac{1}{3}x &= -2 \\ x &= -6 \end{aligned}$$

This result does not satisfy the inequality $x > -2$. Therefore, there are $\boxed{\text{no solutions.}}$

6.4a Problem: Let $f(x) = x + |2x - 1|$. Find all solutions to the equation $f(x) = 8$.

6.4a Solution: We can rewrite this as $x + |2x - 1| = 8 \to |2x - 1| = 8 - x$. There are two equations that need to be solved for:

$$\begin{aligned} 2x - 1 &= 8 - x \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

One solution is $x = 3$.

$$\begin{aligned} 2x - 1 &= -(8 - x) \\ 2x - 1 &= -8 + x \\ x &= -7 \end{aligned}$$

Another solution is $x = -7$. Thus, the two solutions are $\boxed{x = 3 \text{ and } x = -7}$.

6.4b Problem: Let $g(x) = 3x - 3 + |x + 5|$. Find all values of a which satisfy the equation $g(a) = 2a + 8$.

6.4b Solution: $g(a) = 3a - 3 + |a + 5|$; thus the equation becomes $3a - 3 + |a + 5| = 2a + 8$. We can rewrite this as $|a + 5| = 2a + 8 - 3a + 3$. We need to solve for two equations:

$$\begin{aligned} a + 5 &= 2a + 8 - 3a + 3 \\ a + 5 &= -a + 11 \\ 2a &= 6 \\ a &= 3 \end{aligned}$$

Thus, one solution is $a = 3$.

$$\begin{aligned} a + 5 &= -(2a + 8 - 3a + 3) \\ a + 5 &= -(-a + 11) \\ a + 5 &= a - 11 \\ 0 &\neq -16 \end{aligned}$$

This statement is not true; thus the only solution is $\boxed{a = 3.}$

6.4c Problem: Let $h(x) = |x| - 3x + 4$. Find all solutions to the equation $h(x-1) = x - 2$.

6.4c Solution: $h(x-1) = |x-1| - 3(x-1) + 4 = |x-1| - 3x + 7$; thus the equation becomes $|x-1| - 3x + 7 = x - 2$, which can be rewritten as $|x-1| = x - 2 + 3x - 7$. We need to solve for two equations:

$$\begin{aligned} x - 1 &= x - 2 + 3x - 7 \\ x - 1 &= 4x - 9 \\ -3x &= -8 \\ x &= \frac{8}{3} \end{aligned}$$

Thus, one solution is $a = 3$.

$$\begin{aligned} x - 1 &= -(x - 2 + 3x - 7) \\ x - 1 &= -(4x - 9) \\ x - 1 &= -4x + 9 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

When plugging in $x = 2$, the result is $|2-1| - 3(2-1) + 4 = 2$, when the results *should be* $x - 2 \rightarrow 2 - 2 = 0$. Therefore, the only solution is $\boxed{x = \frac{8}{3}.}$