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Ande

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6.1 Context: The absolute value function is defined by a multipart rule. The graph of the absolute value function is pictured below.

6.1a Problem: Calculate: $|0|$, $|2|$, $|-3|$.

6.1a Solution:

- $|0| = 0$
- $|2| = 2$
- $|-3| = 3$

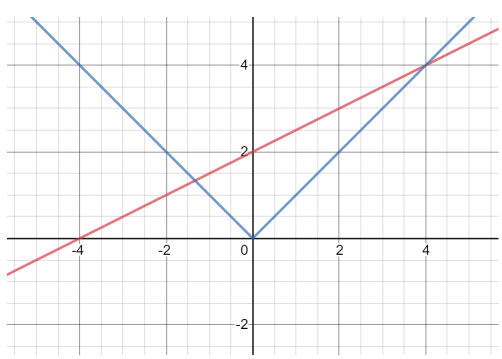
6.1b Problem: Solve for x: $|x| = 4$; $|x| = 0$, $|x| = -1$.

6.1b Solution:

- $|x| = 4$: x can be either 4 or -4.
- $|x| = 0$: x must be 0.
- $|x| = -1$: There are no solutions of x.

6.1c Problem: Sketch the graph of $y = \frac{1}{2}x + 2$ and $y = |x|$ in the same coordinate system. Find where the two graphs intersect, label the coordinates of these point(s), then find the area of the region bounded by the two graphs.

6.1c Solution: The graph is as below with $y = \frac{1}{2}x + 2$ in red and $y = |x|$ in blue. We need to solve for the intersection of the line $y = \frac{1}{2}x + 2$ and $y = x$ and the intersection of the line $y = \frac{1}{2}x + 2$ and $y = -x$, since these are the components of the $y = |x|$ function. The third vertex is simply $(0, 0)$.



Finding one intersection point:

$$
\begin{aligned}
x &= \frac{1}{2}x + 2 \\
\frac{1}{2}x &= 2 \\
x &= 4 \\
y &= 4
\end{aligned}
$$

Thus, one point of the formed triangle is $\boxed{(4,4)}$. Finding the second intersection point:

$$
\begin{aligned}
-x &= \frac{1}{2}x + 2 \\
-\frac{3}{2}x &= 2 \\
x &= -\frac{4}{3} \\
y &= |-\frac{4}{3}| = \frac{4}{3}
\end{aligned}
$$

Thus, the second point of the triangle is $\boxed{\left(-\frac{4}{3}, \frac{4}{3}\right)}$; the third is $(0,0)$. Using the shoelace algorithm, we can write the coordinates in shoelace form:

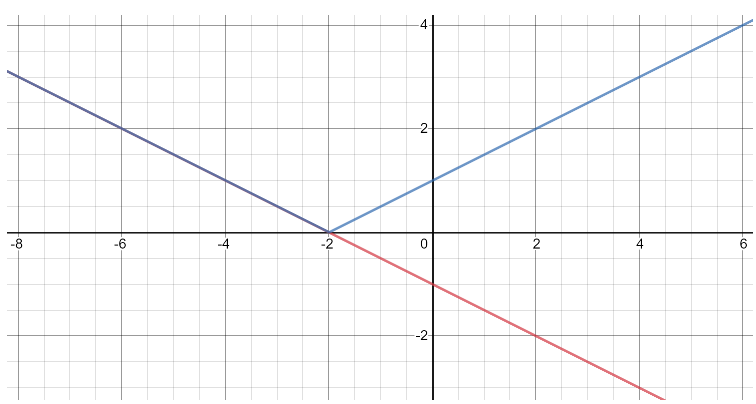
$$
\begin{array}{c|c}
0 & 0 \\
-\frac{4}{3} & \frac{4}{3} \\
4 & 4 \\
0 & 0
\end{array}
$$

Cross multiplying and summing from top left to bottom right yields $-\frac{16}{3}$, cross multiplying and summing from top right to bottom left yields $\frac{16}{3}$. The difference between the two sums is $\frac{32}{3}$. Dividing by two yields an area of $\boxed{\frac{16}{3} \text{ square units}}$.

6.2 Context: For each of the following functions, graph $f(x)$ and $g(x) = |f(x)|$, and give the multipart rule for $g(x)$.

6.2a Problem: $f(x) = -0.5x - 1$

6.2a Solution: The graph, with $g(x)$ in blue and $f(x)$ in red:

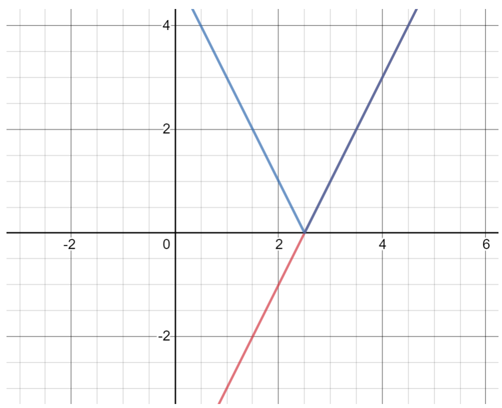


The 'vertex' of the $g(x)$ is simply the x-intercept of $f(x)$, which is $0 = -0.5x - 1 \rightarrow x = -2$. The line extending from $(-2, 0)$ to the right of $g(x)$ is the line of $f(x)$, reflected over the x axis. Thus, its slope is simply the negative of that of $f(x)$, which comes out to be 0.5, and the reflection of the y-intercept from $(0, -1)$ over the x-axis is $(0, 1)$. Thus the equation of the right segment is $0.5x + 1$.

$$g(x) = \begin{cases} -0.5x - 1 & \text{if } x < -2 \\ 0.5x + 1 & \text{if } x \geq -2 \end{cases} \tag{1}$$

6.2b Problem: $f(x) = 2x - 5$

6.2b Solution: The graph, with $g(x)$ in blue and $f(x)$ in red:

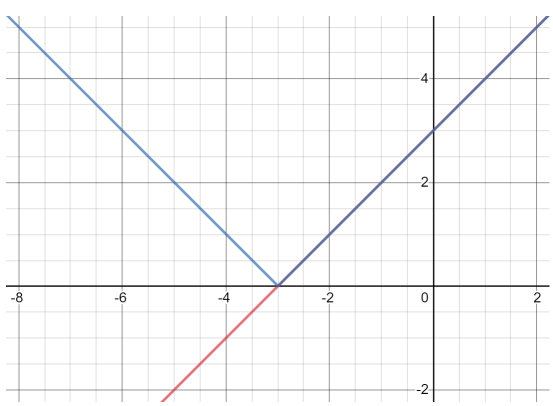


The 'vertex' of the $g(x)$ is simply the x-intercept of $f(x)$, which is $0 = 2x - 5 \rightarrow x = \frac{5}{2}$. The line extending from $(-2.5, 0)$ to the left of $g(x)$ is the line of $f(x)$, reflected over the x axis. Thus, its slope is simply the negative of that of $f(x)$, which comes out to be -2, and the reflection of the y-intercept from $(0, -5)$ over the x-axis is $(0, 5)$. Thus the equation of the right segment is $-2x + 5$.

$$g(x) = \begin{cases} 2x - 5 & \text{if } x > 2.5 \\ -2x + 5 & \text{if } x \leq 2.5 \end{cases} \tag{2}$$

6.2c Problem: $f(x) = x + 3$

6.2b Solution: The graph, with $g(x)$ in blue and $f(x)$ in red:



The 'vertex' of the $g(x)$ is simply the x-intercept of $f(x)$, which is $0 = x + 3 \rightarrow x = -3$. The line extending from $(-3, 0)$ to the left of $g(x)$ is the line of $f(x)$, reflected over the x axis. Thus, its slope is simply the negative of that of $f(x)$, which comes out to be -1, and the reflection of the y-intercept from $(0, 3)$ over the x-axis is $(0, -3)$. Thus the equation of the right segment is $-x - 3$.

$$g(x) = \begin{cases} x + 3 & \text{if } x > 2.5 \\ -x - 3 & \text{if } x \leq 2.5 \end{cases} \tag{3}$$