Collingwood 22

Andre Ye

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5.7 Context: After winning the lottery, you decide to buy your own island. The island is located 1 km offshore from a straight portion of the mainland. There is currently no source of electricity on the island, so you want to run a cable from the mainland to the island. An electrical power sub-station is located 4 km from your island's nearest location to the shore. It costs $50,000 per km to lay a cable in the water and $30,000 per km to lay a cable over the land.

5.7a Problem: Explain why we can assume the cable follows the path indicated in the picture; i.e. explain why the path consists of two line segments, rather than a weird curved path AND why it is OK to assume the cable reaches shore to the right of the power station and the left of the island.

5.7a Solution: The path likely consists of two line segments because it is the shortest way to represent the cable-on-land and cable-on-water possibilities. A weird curved path would be unnecessarily adding more cable than necessary (and would probably violate zoning laws). We can assume the that the cable reaches shore right of the power station and left of the island because the island is right of the power station, and hence the entire length of the cable is between these two locations. The cable reaches shore somewhere along its length, so it must reach shore to the right of the power station and the left of the island.

5.7b Problem: Let x be the distance down-shore from the power sub-station to where the cable reaches the land. Find a function $f(x)$ in the variable x that computes the cost to lay a cable out to your island.

5.7b Solution: It costs $\$30,000$ per kilometer to lay cable across land and $\$50,000$ per kilometer to lay cable across water. x is the distance of cable on land, so that component will cost $\$30,000 \cdot x$, and the remaining distance to the island is $4-x$ kilometers in horizontal (east-west) length and 1 kilometer in vertical (south-north) length. Thus the distance of sea cable is $\$50,000 \cdot \sqrt{(4-x)^2+1}$. The total cost of cable is the sum of these two prices; therefore $\boxed{f(x) = 30,000x + 50,000 \cdot \sqrt{(4-x)^2+1}}$.

5.7c Problem: Make a table of values of $f(x)$, where $x = 0, \frac{1}{2}, 1, \frac{3}{2}, ..., \frac{7}{2}, 4$. Use these calculations to estimate the installation of minimal cost.

x	$f(x)$
0	≈ 206155.28
0.5	≈ 197002.75
1	≈ 188113.88
1.5	≈ 179629.12
2	≈ 171803.40
2.5	≈ 165138.78
3	≈ 160710.68
3.5	≈ 160901.70
4	170000

The minimum cost observed is at $x = 3$; thus the minimum cost is somewhere from $\boxed{2.5 < x < 3.5}$.

5.8 Context: This problem deals with the "mechanical aspects" of working with the rule of a function. For each of the functions listed in (a)-(c), calculate: $f(0)$, $f(-2)$, $f(x+3)$, $f(\heartsuit)$, $f(\heartsuit + \triangle)$.

5.8a Problem: The function $f(x) = \frac{1}{2}(x-3)$ on the domain of all real numbers.

5.8a Solution:

x	$f(x)$
0	$-\frac{3}{2}$
-2	$-\frac{5}{2}$
$x+3$	$\frac{1}{2}(x+3-3) = \frac{1}{2}x$
$\heartsuit$	$\frac{1}{2}(\heartsuit - 3)$
$\heartsuit + \triangle$	$\frac{1}{2}(\heartsuit + \triangle - 3)$

5.8b Problem: The function $f(x) = 2x^2 - 6x$ on the domain of all real numbers.

5.8b Solution:

x	$f(x)$
0	0
-2	20
$x+3$	$2(x+3)^2 - 6(x+3) = 2x^2 + 6x$
$\heartsuit$	$2\heartsuit^2 - 6\heartsuit$
$\heartsuit + \triangle$	$2(\heartsuit + \triangle)(\heartsuit + \triangle - 3)$

Work for last input:

$$
\begin{align}
& 2(\heartsuit + \triangle)^2 - 6(\heartsuit + \triangle) \tag{1} \\
&= 2\heartsuit^2 + 4\heartsuit\triangle + 2\triangle^2 - 6\heartsuit - 6\triangle \tag{2} \\
&= 2\left(\heartsuit^2 + 2\heartsuit\triangle + \triangle^2 - 3\heartsuit - 3\triangle\right) \tag{3} \\
&= 2\left((\heartsuit + \triangle) \cdot (\heartsuit + \triangle) - 3(\heartsuit + \triangle)\right) \tag{4} \\
&= 2(\heartsuit + \triangle)(\heartsuit + \triangle - 3) \tag{5}
\end{align}
$$

5.8c Problem: The function $f(x) = 4\pi^2$.

x	$f(x)$
0	$4\pi^2$
-2	$4\pi^2$
$x+3$	$4\pi^2$
$\heartsuit$	$4\pi^2$
$\heartsuit + \triangle$	$4\pi^2$

5.9 Problem: Which of the curves in Figure 5.14 represent the graph of a function? If the curve is not the graph of a function, describe what goes wrong and how you might "fix it." When you describe how to "fix" the graph, you are allowed to cut the curve into pieces and such that each piece is the graph of a function. Many of these problems have more than one correct answer.

5.9 Solution:

Graph	Function?	Fix (if applicable)
a	No	Take only the top or bottom half of the ellipse.
b	No	Eliminate the portion below the x-axis.
c	Yes	
d	No	Take only the top two line segments or the bottom two line segments.
e	Yes	
f	No	Remove the loop at the top.
g	Yes	
h	No	Remove the vertical components on the sides.
i	Yes	
j	No	Take only the top or bottom parts of the shape.
k	No	Take only the top or bottom segments of the square as the function.
l	No	Take only top and bottom semicircles of the circle or remove the full circle completely.
m	No	When two points have the same x value, eliminate one of them.
n	No	Remove the loops and choose only the top or bottom segment as the function.
o	Yes	
p	Yes	