Collingwood 21

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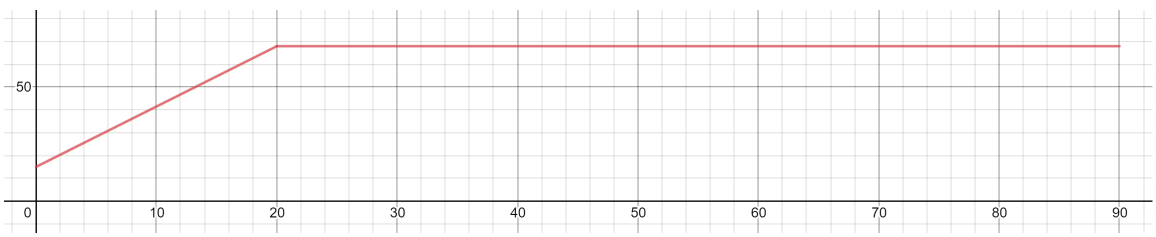
5.5 Context: Sketch a reasonable graph for each of the following functions. Specify a reasonable domain and range and state any assumptions you are making. Finally, describe the largest and smallest values of your function.

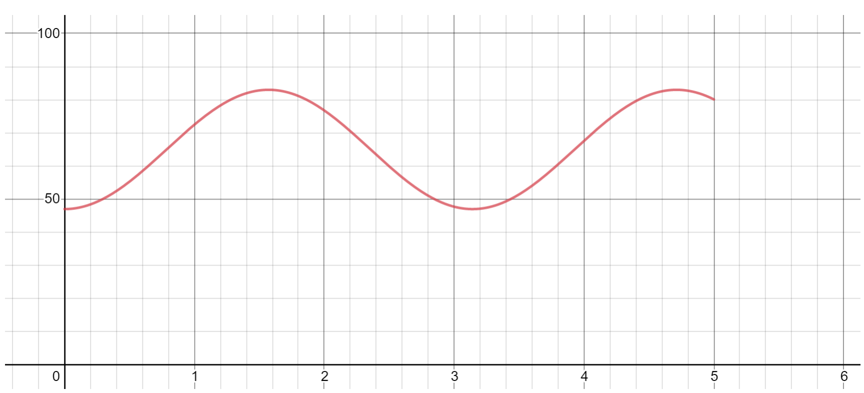
5.5a Question: Height of a person depending on age.

5.5a Solution: A newborn is about 15 inches long; the height of an adult is about 68 inches, and most stop growing after 20. Thus, we have two points (0, 15) and (20, 68). This portion can be modeled with the equation $y = \frac{53}{20}x + 15$ from $0 \leq x \leq 20$, where x represents the age in years and y represents the height, in inches. After this, to $x = 90$, the number of years a human would aspire to live; thus the graph is $y = 68$ for $20 < x \leq 90$. We assume that growth is linear until 20 years old, and that no growth happens after then. The graph is bounded from 0 years (birth) to 90 years (passing away). The largest value that can be achieved is $y = 68$, and the smallest value is $y = 15$.



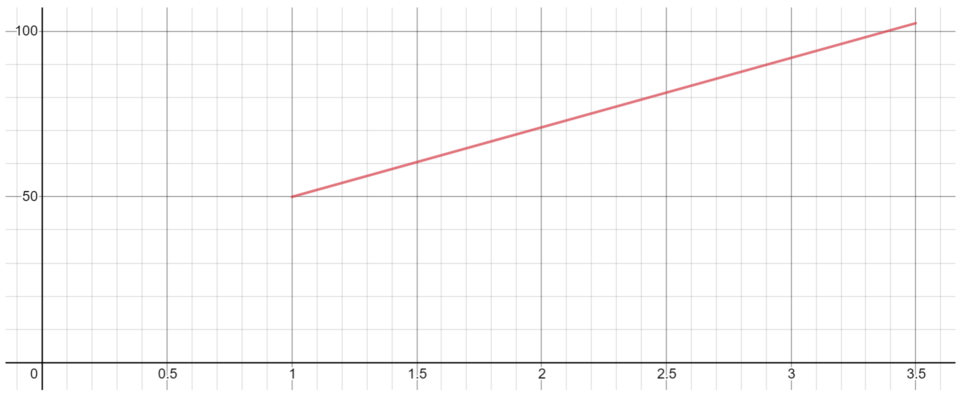
5.5b Question: Height of the top of your head as you jump on a pogo stick for 5 seconds.

5.5b Solution: If I jump on a pogo stick for 5 sec, it is sinusoidal. My height is 5 feet 5 inches, so I begin at 65 inches above the ground. Assuming a jump can carry me 3 feet, or 36 inches, into the air, the graph can be represented by $y = 65 - 18\cos(2x)$, where y is the height in inches of the top of my head after x seconds. We make assumptions about the frequency and fixed height of pogo jumps. We also assume that we begin at the lowest possible height. The domain is restricted by 5 seconds, the length of the duration outlined in the question. The smallest value is $y = 65$ and the largest value is $y = 101$.



5.5c Question: The amount of postage you must put on a first class letter, depending on the weight of the letter.

5.5c Solution: The first rate for one ounce letters is 50 cents, then 21 cents for each additional ounce. This can be represented by $y = 21(x-1)+50$, where x is the number of ounces of the letter and y is the cost, in cents. The maximum limit for a letter is 3.5 ounces, so the domain is restricted by $1 \leq x \leq 3.5$. We make assumptions about decimal values of ounces existing. The domain is restricted by 1 ounce to 3.5 ounces, the minimum and maximum, respectively. The minimum value is $y = 21$ and the maximum value is $y = 102.5$.

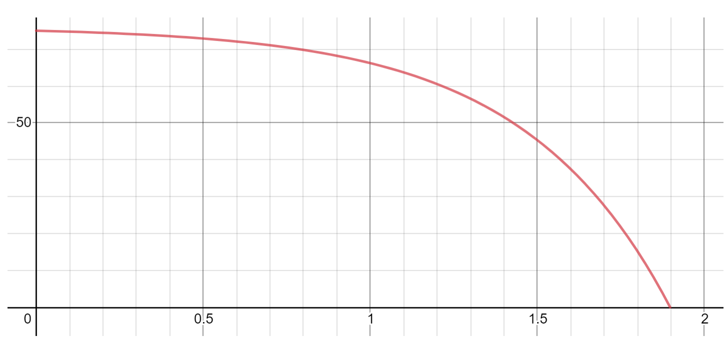


5.5d Problem: Distance of your big toe from the ground as you ride your bike for 10 seconds.

5.5d Solution: Our big toe would begin at probably about 5 inches from the ground initially, and is 23 inches long at its maximum height. Thus, it can be represented with $y = 14 - 9\cos(2x)$. We make assumptions about the height of the wheel and that we pedal at a constant weight. We also assume our big toe begins on the downpedal at time zero. Our domain is restricted by 10 seconds, as outlined by the problem. The minimum value is $y = 5$ and the maximum is $y = 23$.

5.5e Problem: Your height above the water level in a swimming pool after you dive off the high board.

5.5e Solution: Let the high board be 75 feet tall. It can be represented as $y = -\frac{1}{9.8^{-x}} + 76$, where y represents the height in feet of the diver above the water level and x represents the number of seconds that have elapsed. We assume that the relationship of height is exponential. We also assume that the diver does not jump before they begin falling. The graph is bounded by $x \geq 0$ and $y \geq 0$, since any other value would be impractical. The minimum value is $y = 0$ and the maximum value is $y = 75$.



5.6 Context: Here is a picture of the graph of the function $f(x) = 3x^2 - 3x - 2$.

5.6a Problem: Find the x and y intercepts of the graph.

5.6a Solution: To find the y-intercept, we can simply find the value when $x = 0$; this comes out to be $\boxed{(0, 2)}$. To find the x-intercept, we need to find values for x for which $f(x) = 0$.

$$\begin{aligned} 3x^2 - 3x - 2 &= 0 \\ x &= \frac{3 \pm \sqrt{9 - 4(3)(-2)}}{6} \\ x &= \frac{3 \pm \sqrt{33}}{6} \end{aligned}$$

Thus, the x-intercepts are $\boxed{\left(\frac{3 \pm \sqrt{33}}{6}, 0\right)}$.

5.6b Problem: Find the exact coordinates of all points (x,y) on the graph which have y-coordinate equal to 5.

5.6b Solution: To answer this problem, we must find values of x such that $f(x) = 5$.

$$\begin{aligned} 3x^2 - 3x - 2 &= 5 \\ 3x^2 - 3x - 7 &= 0 \\ x &= \frac{3 \pm \sqrt{9 - 4(3)(-7)}}{6} \\ x &= \frac{3 \pm \sqrt{93}}{6} \end{aligned}$$

Thus, the coordinates that satisfy this are $\boxed{\left(\frac{3 \pm \sqrt{93}}{6}, 5\right)}$.

5.6c Problem: Find the coordinates of all points (x,y) on the graph which have y-coordinate equal to -3.

5.6c Solution: To answer this problem, we must find values of x such that $f(x) = -3$.

$$\begin{aligned} 3x^2 - 3x - 2 &= -3 \\ 3x^2 - 3x + 1 &= 0 \\ x &= \frac{3 \pm \sqrt{9 - 4(3)(1)}}{6} \\ x &= \frac{3 \pm \sqrt{-3}}{6} \end{aligned}$$

Thus, $\boxed{\text{there are no real coordinates that satisfy this condition.}}$

5.6d Problem: Which of these points is on the graph: $(1, -2)$, $(-1, 3)$, $(2.4, 8)$, $(\sqrt{3}, 7 - 3\sqrt{3})$.

5.6d Solution: We can evaluate the x values and see if they correspond to the y-values.

- Testing $(1, -2)$. $3x^2 - 3x - 2 = 3(1) - 3 - 2 = -2$. This point is on the graph.
- Testing $(-1, 3)$. $3x^2 - 3x - 2 = 3(1) + 3 - 2 = 4$. This point is not on the graph.
- Testing $(2.4, 8)$. $3x^2 - 3x - 2 = 3(5.76) - 3(2.4) - 2 = 8.08$. This point is not on the graph.

- Testing $(\sqrt{3}, 7-3\sqrt{3})$. $3x^2-3x-2 = 3(3)-3\sqrt{3}-2 = 7-3\sqrt{3}$. This point is on the graph.

Thus, the points $\boxed{(1,-2) \text{ and } \left(\sqrt{3}, 7-3\sqrt{3}\right) \text{ are on the graph.}}$

5.6e Problem: Find the exact coordinates of the point (x,y) on the graph with $x = \sqrt{1+\sqrt{2}}$.

5.63 Solution: Plugging in this value of x yields $3(\sqrt{1+\sqrt{2}})^2 - 3(\sqrt{1+\sqrt{2}}) - 2 = 3(\sqrt{1+\sqrt{2}}) - 3(\sqrt{1+\sqrt{2}}) - 2$. Thus, the exact coordinate of the point is $\boxed{\left(\sqrt{1+\sqrt{2}}, 3\left(1+\sqrt{2}\right) - 3\left(\sqrt{1+\sqrt{2}}\right) - 2\right)}$.