Collingwood 20

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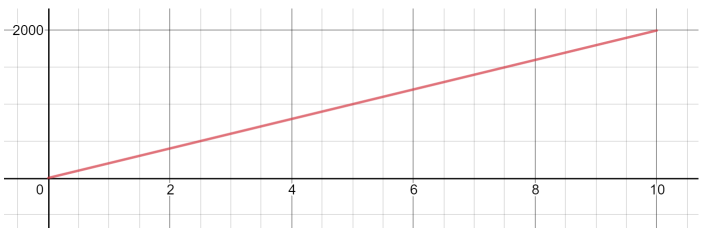
5.3 Context: Dave leaves his office in Padelford Hall on his way to teach in Gould Hall. Below are several different scenarios. In each case, sketch a plausible (reasonable) graph of the function $s = d(t)$ which keeps track of Dave's distance s from Padelford Hall at time t. Take distance units to be "feet" and time units to be "minutes." Assume Dave's path to Gould Hall is along a straight line which is 2400 feet long.

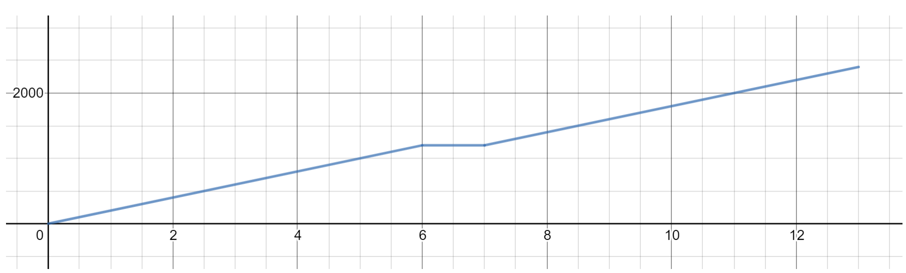
5.3a Problem: Dave leaves Padelford Hall and walks at a constant speed until he reaches Gould Hall 10 minutes later.

5.3a Solution: If Dave walks at a constant speed and takes 10 minutes to reach Gould Hall, then at time t (in minutes) he is $\frac{2400}{10}t = 240t$ feet away from Padelford Hall. Thus, the equation can be represented as $s = 240t$ with domain restriction $0 \leq t \leq 10$.



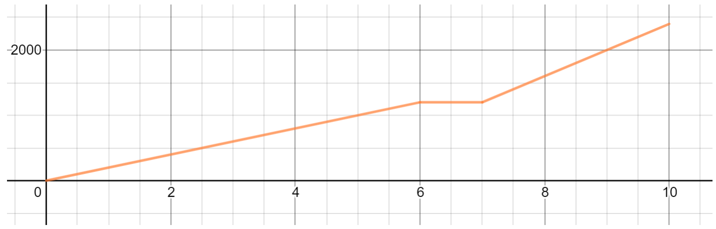
5.3b Problem: Dave leaves Padelford Hall and walks at a constant speed. It takes him 6 minutes to reach the half-way point. Then he gets confused and stops for 1 minute. He then continues on to Gould Hall at the same constant speed he had when he originally left Padelford Hall.

5.3b Solution: The halfway point is $\frac{2400}{2} = 1200$ feet away from Padelford; Dave takes 6 minutes to cover this distance. Thus he walks at a speed of $\frac{1200}{6} = 200$ feet per minute. For the domain range $0 \leq t \leq 6$, the graph can be represented as $s = 200t$. Because he pauses for 1 minute, from $6 < t \leq 7$, $s = 1200$ (where he was when he stopped walking at 6 minutes). We can represent the rightmost point of this as $(7, 1200)$. Dave continues walking at a speed of 200 feet per minute from this point, thus its equation can be represented in point-slope form as $s - 1200 = 200(t - 7) \rightarrow s = 200t - 200$. Thus, from $7 < t \leq 13$, $s = 200t - 200$.



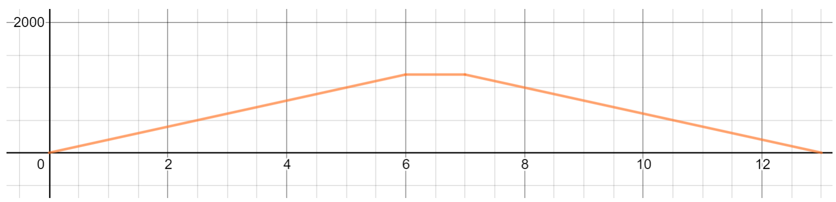
5.3c Problem: Dave leaves Padelford Hall and walks at a constant speed. It takes him 6 minutes to reach the half-way point. Then he gets confused and stops for 1 minute to figure out where he is. Dave then continues on to Gould Hall at twice the constant speed he had when he originally left Padelford Hall.

5.3c Solution: Since this problem has the same parameters as 5.3b, we have already calculated that through domain restriction $0 \leq t \leq 6$, the graph can be represented as $s = 200t$ and from $6 < t \leq 7$, $s = 1200$. However, after a 1 minute episode of confusion, Dave walks two times as fast in this problem at $200 \cdot 2 = 400$ feet per minute. Thus, he travels the remaining 1200 feet in $\frac{1200}{400} = 3$ minutes. We can represent this equation as $s - 1200 = 400(t-7) \rightarrow s = 400t - 1600$ with domain restriction $7 < t < 10$.



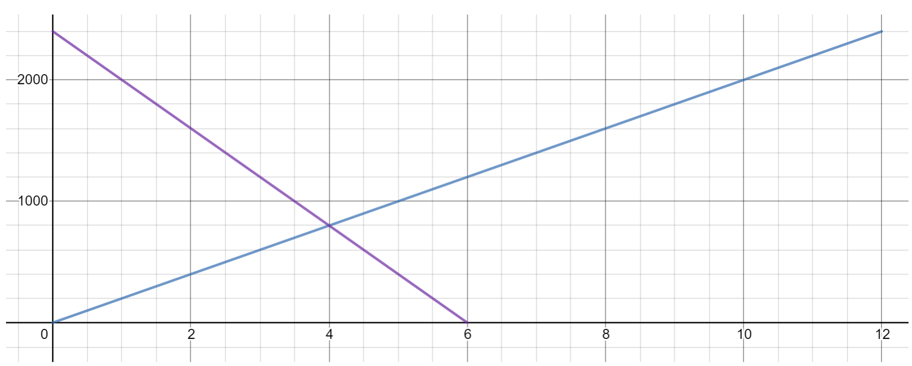
5.3d Problem: Dave leaves Padelford Hall and walks at a constant speed. It takes him 6 minutes to reach the half-way point. Dave gets confused and stops for 1 minute to figure out where he is. Dave is totally lost, so he simply heads back to his office, walking the same constant speed he had when he originally left Padelford Hall.

5.3d Solution: Since this problem has the same parameters as 5.3b, we have already calculated that through domain restriction $0 \leq t \leq 6$, the graph can be represented as $s = 200t$ and from $6 < t \leq 7$, $s = 1200$. However, after a 1 minute episode of confusion, Dave returns to the office at the same speed of 200 feet per minute; thus his equation is $s - 1200 = -200(t-7) \rightarrow s = -200t + 2600$ with domain restriction $7 < t \leq 13$.



5.3e Problem: Dave leaves Padelford heading for Gould Hall at the same instant Angela leaves Gould Hall heading for Padelford Hall. Both walk at a constant speed, but Angela walks twice as fast as Dave. Indicate a plot of "distance from Padelford" vs. "time" for both Angela and Dave.

5.3e Solution: Dave walks at a constant speed of 200 feet per minute; thus he travels the distance in $\frac{2400}{200} = 12$ minutes. Angela walks at $200 \cdot 2 = 400$ feet per minute; thus she travels the distance in $\frac{2400}{400} = 6$ minutes. Their two equations are thus $s = 200t$ with domain restriction $0 \leq t \leq 12$ and $s = 2400 - 400t$ with domain restriction $0 \leq t \leq 6$, respectively. In Angela's equation, she begins at location (0,2400) (2400 feet away from Padelford Hall) and walks 400 feet closer to Padelford hall every minute; thus an equation representing her path over time has y-intercept 2400 and slope -400. *Below: Angela in purple and Dave in blue.*



5.3f Problem: Suppose you want to sketch the graph of a new function $s = g(t)$ that keeps track of Dave's distance s from Gould Hall at time t. How would your graphs change in (a)-(e)?

5.3f Solution: The distance of Dave from Gould Hall is simply 2400 minus the distance of Dave from Padelford Hall (and the same applies vice versa). Specifically, one mathematical definition would be $g(t) = -d(t) + 2400$; we reflect one function over the x-axis to 'flip' it, then translate it upwards 2400 feet such that it begins at $(0, 2400)$, which is where Dave should begin (the maximum distance away from Gould Hall).

5.4 Context: At 5 AM one day, a monk began a trek from his monastery by the sea to the monastery at the top of a mountain. He reached the mountain-top monastery at 11 AM, spent the rest of the day in meditation, and then slept the night there. In the morning, at 5 AM, he began walking back to the seaside monastery. Though walking downhill should have been faster, he dawdled in the beautiful sunshine, and ending up getting to the seaside monastery at exactly 11 AM.

5.4a Problem: Was there necessarily a time during each trip when the monk was in exactly the same place on both days? Why or why not?

5.4a Solution: $\boxed{\text{Yes,}}$ we can imagine the monk as travelling along a line $\overline{AB}$ from point A (his starting point) to point B (his ending point, the monastery). Let there be two versions of the monk, J and K, which begin at points A and B, respectively, and move towards the other point at the same constant speed. This represents the monk's travel on different days. J and K must meet at the same time and at the same location.

5.4b Problem: Suppose the monk walked faster on the second day, and got back at 9 AM. What is your answer to part (a) in this case?

5.4b Solution: $\boxed{\text{The answer does not change.}}$ The question simply asks whether monks J and K, as outlined in 5.4a, will meet each other. As long as one or more of the speeds is not zero, they will always meet each other before they reach the opposite point, and hence be in the same place at the same time.

5.4c Problem: Suppose the monk started later, at 10 AM, and reached the seaside monastery at 3 PM. What is your answer to part (a) in this case?

5.4c Solution: They will always be, somewhere along their path, at the same point at the same time, as long as at least one monk is moving along the line and the line is bounded. The answer will not change in any scenario unless both monks have speed 0 (do not move). Therefore, the $\boxed{\text{answer does not change.}}$

As an additional more formal proof, assuming the monk walks at a constant speed (which generalizes to other speed patterns): Let x represent the number of Planck lengths the monks are away from the initial monastery by the sea (x_J as Monk J's location, x_K as Monk K's location). Let the monks travel at 1 Planck length per hour. (As long as the movements of each of the monks on this coordinate system are at an equal constant speed, the absolute value is arbitrary.) Thus, Monk J begins at $x = 0$ and ends at $x = 6$; let t be the number of hours from 5 AM. We can write Monk J's position as $x_J = t$. Monk K begins at $x = 6$ and ends at $x = 0$; we can write Monk K's position as $x_K = -(t-5) + 6$. If monks J and K meet, then there can be a solution to $x_J = x_K$ with a time t that is less than or equal to 6. This comes to be $t = -(t-5) + 6 \rightarrow 5.5$, which is ≤ 6.