Collingwood Homework 2

Andre Ye

6 October 2020

1.4 Question: The Eiffel Tower has a mass of 7.3 million kilograms and a height of 324 meters. Its base is square with a side length of 125 meters. The steel used to make the Tower occupies a volume of 930 cubic meters. Air has a density of 1.225 kg per cubic meter. Suppose the Tower was contained in a cylinder. Find the mass of the air in the cylinder. Is this more or less than the mass of the Tower?

1.4 Solution: If the tower with a square base was contained in a cylinder, the square base would be inscribed inside the base of the cylinder (which will be referred to as "a circle"). Because of this, the diameter of the circle is equal to the diagonal of the square.

Because the diagonal of a square with side length s is $s\sqrt{2}$, the diagonal of a square with side length 125 meters is $125\sqrt{2}$ meters. Hence, the diameter of the circle is $125\sqrt{2}$ meters, and correspondingly its radius - half the length of the diameter - is $\frac{125}{2}\sqrt{2}$ meters.

Because the area of a circle with radius r is πr^2, the area of the circle can be found by plugging in the derived radius:

$$A = \pi\left(\frac{125}{2}\sqrt{2}\right)^2 = \pi\left(\frac{15,625}{4}\cdot 2\right) = \pi\left(\frac{15,625}{2}\right) = \frac{15,625}{2}\pi \text{ meters}^2$$

The Eiffel Tower has a height of 324 meters; therefore the height of a cylinder enclosing it will also be 324 meters. The volume of a cylinder can be given by $A \cdot H$, where A is the area of the base and H is the height of the cylinder:

$$V_{\text{cylinder}} = \frac{15,625}{2}\pi \cdot 324 = 2,531,250\pi \text{ meters}^3$$

Before the mass of the air is calculated, the volume of the steel used to build the Eiffel Tower (930 meters^3) needs to be subtracted from V_{cylinder}, the volume of a cylinder enclosing the tower.

$$V_{\text{air}} = 2,531,250\pi - 930 \text{ meters}^3$$

The mass of air is 1.225 kg per cubic meter. This means that for each cubic meter of air, the mass grows by 1.225 kg. Then, the mass is $(2,531,250\pi - 930)$ meters$^3 \cdot 1.225\frac{\text{kg}}{\text{meter}^3} \approx 9,740,252.3$ kg.

The mass of the Eiffel Tower is 7.3 million kilograms, or $7,300,000$ kg. This is less than than the mass of the air within the enclosing cylinder, $\approx 9,740,252.3$ kg.

Therefore, the $\boxed{\text{the mass of the air is larger than the mass of the Tower.}}$

1.5a Question: Lee has a speed of 16 ft/sec; what is his pace?

1.5a Solution: Pace is given as the number of minutes it takes to complete one mile. Lee runs 16 feet per second. First, one must convert Lee's speed into miles per minute.

There are 60 seconds in a minute. Therefore, if Lee runs 16 feet per second, then he runs $16 \cdot 60 = 960$ feet in one minute. Because there are 5280 feet in one mile, Lee runs $\frac{960}{5280}$ miles per minute.

Using the distance = speed $\cdot$ time framework, one can find the pace - how long Lee takes to complete one mile. The distance is replaced with 1 (one mile), and the speed is replaced with $\frac{960}{5280}$, Lee's speed.

$$1 = \frac{960}{5280} \cdot \text{time}$$
$$\frac{5280}{960} = \text{time}$$
$$\text{time} = 5.5 \text{ minutes}$$

Hence, Lee's pace is $\boxed{5.5 \text{ minutes/mile.}}$

1.5b Question: Allyson has a pace of 6 min/mile; what is her speed?

1.5b Solution: If Allyson's pace is 6 minutes per mile, then Allyson can complete one mile in six minutes. Thus, the following $d = s \cdot t$ framework can be constructed:

$$5280 \text{ feet} = s \text{ ft/s} \cdot 360 \text{ seconds}$$
$$s = \frac{5280}{360} = 14\frac{2}{3} \text{ ft/s}$$

Therefore, Allyson's speed is $\boxed{14\frac{2}{3} \text{ ft/s.}}$

1.5c Question: Adrienne and Dave are both running a race. Adrienne has a pace of 5.7 min/mile and Dave is running 10.3 mph. Who is running faster?

1.5c Answer: To compare, let use convert Adrienne's pace into mile per hour. If Adrienne's pace is 5.7 min/mile, she can run one mile in 5.7 minutes.

$$1 \text{ mile} = s \text{ mph} \cdot \frac{5.7}{60} \text{ hours}$$
$$s \approx 10.53 \text{ mph}$$

Since 10.53 mph $>$ 10.3 mph, $\boxed{\text{Adrienne is running faster.}}$