Collingwood 19

Andre Ye

November 2020

5.1 Context: For each of the following functions, find the expression for $\frac{f(x+h)-f(x)}{h}$. Simplify each of your expressions far enough so that plugging in $h = 0$ would be allowed.

5.1a Problem: $f(x) = x^2 - 2x$.

5.1a Solution:

$$\begin{aligned} f(x) &= x^2 - 2x \\ \frac{f(x+h)-f(x)}{h} &= \frac{\left((x+h)^2 - 2\left(x+h\right)\right) - \left(x^2 - 2x\right)}{h} \\ &= \frac{h^2 + 2hx - 2h}{h} \\ &= h + 2x - 2 \end{aligned}$$

Thus, the simplified solution is $\boxed{h + 2x - 2}$.

5.1b Problem: $f(x) = 2x + 3$.

5.1b Solution:

$$\begin{aligned} f(x) &= 2x + 3 \\ \frac{f(x+h)-f(x)}{h} &= \frac{(2\left(x+h\right) + 3) - (2x + 3)}{h} \\ &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

Thus, the simplified solution is $\boxed{2}$.

5.1c Problem: $f(x) = x^2 - 3$.

5.1c Solution:

$$\begin{aligned} f(x) &= x^2 - 3 \\ \frac{f(x+h)-f(x)}{h} &= \frac{\left((x+h)^2 - 3\right) - \left(x^2 - 3\right)}{h} \\ &= \frac{h^2 + 2hx}{h} \\ &= h + 2x \end{aligned}$$

Thus, the simplified solution is $\boxed{h + 2x}$.

5.1d Problem: $f(x) = 4 - x^2$.

5.1d Solution:

$$\begin{aligned} f(x) &= 4 - x^2 \\ \frac{f(x+h) - f(x)}{h} &= \frac{\left(4 - (x+h)^2\right) - \left(4 - x^2\right)}{h} \\ &= \frac{-h^2 - 2hx}{h} \\ &= -h - 2x \end{aligned}$$

Thus, the simplified solution is $\boxed{-h - 2x}$.

5.1e Problem: $f(x) = -\pi x^2 - \pi^2$.

5.1e Solution:

$$\begin{aligned} f(x) &= -\pi x^2 - \pi^2 \\ \frac{f(x+h) - f(x)}{h} &= \frac{\left(-\pi (x+h)^2 - \pi^2\right) - \left(-\pi x^2 - \pi^2\right)}{h} \\ &= \frac{-\pi h^2 - 2\pi hx}{h} \\ &= -\pi h - 2\pi x \end{aligned}$$

Thus, the simplified solution is $\boxed{-\pi h - 2\pi x}$.

5.1f Problem: $f(x) = \sqrt{x - 1}$.

5.1f Solution:

$$\begin{aligned} f(x) &= \sqrt{x-1} \\ \frac{f(x+h) - f(x)}{h} &= \frac{\left(\sqrt{(x+h) - 1}\right) - \left(\sqrt{x-1}\right)}{h} \\ &= \frac{\left(\sqrt{(x+h) - 1}\right) - \left(\sqrt{x-1}\right)}{h} \cdot \frac{\left(\sqrt{(x+h) - 1}\right) + \left(\sqrt{x-1}\right)}{\left(\sqrt{(x+h) - 1}\right) + \left(\sqrt{x-1}\right)} \\ &= \frac{\sqrt{(x+h) - 1}\Big)^2 - \left(\sqrt{x-1}\right)^2}{h \cdot \left(\sqrt{(x+h) - 1}\right) + \left(\sqrt{x-1}\right)} \\ &= \frac{h}{h \cdot \left(\sqrt{(x+h) - 1}\right) + \left(\sqrt{x-1}\right)} \\ &= \frac{1}{\sqrt{(x+h) - 1} + \sqrt{x-1}} \end{aligned}$$

Thus, the simplified solution is $\boxed{\frac{1}{\sqrt{x+h-1} + \sqrt{x-1}}}$.

5.2 Problem: Here are the graphs of two linear functions on the domain $0 \leq x \leq 20$. Find the formula for each of the rules $y = f(x)$ and $y = g(x)$. Find the formula for a NEW function $v(x)$ that calculates the vertical distance between the two lines at x. Explain in terms of the picture what $v(x)$ is calculating. What is $v(5)$? What is v(20)? What are the smallest and largest values of $v(x)$ on the domain $0 \leq x \leq 20$?

5.2 Solution: The two lines extend from the point $(0, 24)$ to $(20, 60)$ and from point $(0, 4)$ to point $(20, 20)$. Thus the two slopes of the lines are $\frac{9}{5}$ and $\frac{4}{5}$, respectively; the equations of the two lines come out to be $g(x) = \frac{9}{5}x + 24$ and $f(x) = \frac{4}{5}x + 4$. Thus the distance between the two lines is simply the higher one minus the smaller one; this is $v(x) = g(x) - f(x) = \left(\frac{9}{5}x + 24\right) - \left(\frac{4}{5}x + 24\right) = x + 20$. Thus, $\boxed{v(x) = x + 20}$. In terms of visual representations, $v(x)$ represents the length of a vertical bar between the two lines at a certain location x. $\boxed{v(5) = 25 \text{ and } v(20) = 40}$. The minimum and maximum values of $v(x)$ are at $x = 0$ and $x = 20$ (at the extremes of the domain), respectively. Thus comes to be $\boxed{\text{minimum 20 and maximum 40}}$.