Collingwood 18

Andre

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4.14 Problem: Juliet and Mercutio are moving at constant speeds in the xy-plane. They start moving at the same time. Juliet starts at the point $(0, -6)$ and heads in a straight line toward the point $(10, 5)$, reaching it in 10 seconds. Mercutio starts at $(9, -14)$ and moves in a straight line. Mercutio passes through the same point on the x axis as Juliet, but 2 seconds after she does. How long does it take Mercutio to reach the y-axis?

4.14 Solution: Juliet begins at the point $(0, -6)$ and moves towards $(10, 5)$; thus she travels 10 units right and 11 units up in 10 seconds. This means that she travels 1 unit right and $\frac{11}{10}$ units up every second. As a line, she travels with slope $\frac{5-(-6)}{10-0} = \frac{11}{10}$; thus her path can be represented by $y = \frac{11}{10}x - 6$. Thus she crosses the x-axis at point $0 = \frac{11}{10}x - 6 \rightarrow \frac{11}{10}x = 6 \rightarrow x = \frac{60}{11}$. Her location can be modeled with $(t, -6 + \frac{11}{10}t)$ at every second t. She takes $0 = -6 + \frac{11}{10}t \rightarrow t = \frac{60}{11}$ seconds to reach the y-intercept.

Hence, Mercutio travels from $(9, -14)$ to the point $(\frac{60}{11}, 0)$ 2 seconds after Juliet does, which comes out to be $\frac{60}{11} + 2 = \frac{82}{11}$ seconds. In terms of x-axis movement, Mercutio moves $9 - \frac{60}{11} = \frac{99}{11} - \frac{60}{11} = \frac{39}{11}$ units left; he takes $\frac{82}{11}$ seconds to perform this movement; hence he moves $\frac{39}{11} \cdot \frac{11}{82} = \frac{39}{82}$ units left every second. Thus, we can represent his x-coordinate location as $9 - \frac{39}{82}t$ after t seconds. For him, to reach the y-axis, we must find the point for which his x-location equals 0.

$$\begin{aligned} 9 - \frac{39}{82}t &= 0 \\ \frac{39}{82}t &= 9 \\ t &= 9 \cdot \frac{82}{39} \approx 18.9231 \end{aligned}$$

Thus, Mercutio takes about $\boxed{\text{18.9231 seconds.}}$

4.15a Problem: Solve for x: $\frac{1}{x} - \frac{1}{x+1} = 3$.

4.15a Solution:

$$\begin{aligned} \frac{1}{x} - \frac{1}{x+1} &= 3 \\ 1 - \frac{x}{x+1} &= 3x \\ x + 1 - x &= 3x(x+1) \\ 1 &= 3x^2 + 3x \\ 3x^2 + 3x - 1 &= 0 \\ x &= \frac{-3 \pm \sqrt{9 - 4(3)(-1)}}{2(3)} \\ x &= \frac{-3 \pm \sqrt{21}}{6} \end{aligned}$$

Thus, $\boxed{x = \frac{-3\pm\sqrt{21}}{6}}$.

4.15b Problem: Solve for t: $2 = \sqrt{(1+t)^2 + (1-2t)^2}$.

4.15b Solution:

$$\begin{aligned}
2 &= \sqrt{(1+t)^2 + (1-2t)^2} \\
4 &= (1+t)^2 + (1-2t)^2 \\
4 &= 1 + 2t + t^2 + 1 - 4t + 4t^2 \\
4 &= 5t^2 - 2t + 2 \\
5t^2 - 2t - 2 &= 0 \\
t &= \frac{2 \pm \sqrt{(-2)^2 - 4(5)(-2)}}{2(5)} \\
t &= \frac{2 \pm 2\sqrt{11}}{10}
\end{aligned}$$

Thus, $\boxed{t = \frac{1 \pm \sqrt{11}}{5}}$.

4.15c Problem: Solve for t: $\frac{3}{\sqrt{5}} = \sqrt{(1+t)^2 + (1-2t)^2}$.

4.15c Solution:

$$\begin{aligned}
\frac{3}{\sqrt{5}} &= \sqrt{(1+t)^2 + (1-2t)^2} \\
\frac{9}{5} &= (1+t)^2 + (1-2t)^2 \\
\frac{9}{5} &= 5t^2 - 2t + 2 \\
9 &= 25t^2 - 10t + 10 \\
25t^2 - 10t + 1 &= 0 \\
(5t-1)^2 &= 0 \\
t &= \frac{1}{5}
\end{aligned}$$

Thus, $\boxed{t = \frac{1}{5}}$.

4.15d Problem: Solve for x: $0 = \sqrt{(1+t)^2 + (1-2t)^2}$.

4.15d Solution:

$$\begin{aligned}
0 &= \sqrt{(1+t)^2 + (1-2t)^2} \\
0 &= (1+t)^2 + (1-2t)^2 \\
0 &= 5t^2 - 2t + 2
\end{aligned}$$

The discriminant is $(-2)^2 - 4(5)(2) = -36$. Because the discriminant is negative, $\boxed{\text{there are no real solutions.}}$

4.16a Problem: Solve for x: $x^4 - 4x^2 + 2 = 0$.

4.16a Solution:

$$
\begin{aligned}
x^4 - 4x^2 + 2 &= 0 \\
x^4 - 4x^2 + 4 &= 2 \\
(x^2 - 2)^2 &= 2 \\
x^2 - 2 &= \pm\sqrt{2} \\
x^2 &= \pm\sqrt{2} + 2 \\
x &= \pm\sqrt{\pm\sqrt{2} + 2}
\end{aligned}
$$

Thus, there are four solutions:

- $\sqrt{\sqrt{2} + 2}$
- $\sqrt{-\sqrt{2} + 2}$
- $-\sqrt{\sqrt{2} + 2}$
- $-\sqrt{-\sqrt{2} + 2}$

4.16b Problem: Solve for y: $y - 2\sqrt{y} = 4$.

4.16b Solution:

$$
\begin{aligned}
y - 2\sqrt{y} &= 4 \\
y - 2\sqrt{y} - 4 &= 0 \\
y - 2\sqrt{y} + 1 &= 5 \\
(\sqrt{y} - 1)^2 &= 5 \\
\sqrt{y} - 1 &= \pm\sqrt{5} \\
\sqrt{y} &= \pm\sqrt{5} + 1 \\
y &= (\pm\sqrt{5} + 1)^2 \\
y &= (\pm\sqrt{5})^2 + 2(\pm\sqrt{5}) + 1 \\
y &= 5 + 2(\pm\sqrt{5}) + 1 \\
y &= \pm 2\sqrt{5} + 6
\end{aligned}
$$

$y = -2\sqrt{5} + 6$ is not valid when plugged back into the equation. Thus, the solution is $\boxed{y = 2\sqrt{5} + 6}$.