Collingwood 16

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4.8 Problem: Dave is going to leave academia and go into business building grain silos. A grain silo is a cylinder with a hemispherical top, used to store grain for farm animals. If Dave is standing next to a silo of cross-sectional radius $r = 8$ feet at the indicated position, his vision will be partially obstructed. Find the portion of the y-axis that Dave cannot see.

4.8 Solution: Let (a, b) be the point of intersection between the circle and the tangent line. The slope between the origin of the circle and this point is $\frac{b}{a}$. Since the tangent line must be perpendicular to this, the slope of the tangent line would be $-\frac{a}{b}$. Furthermore, the slope of the tangent line can be calculated as $\frac{b}{a-12}$ (rise over run). These two slopes must be the same, thus we have $-\frac{a}{b} = \frac{b}{a-12} \to b^2 = -a^2 + 12a$.

Let the equation of the circle be represented by $x^2 + y^2 = 64$. a and b are substitutes for the x and y coordinates, thus we can write $x^2 - x^2 + 12x = 64$. Simplifying, $x = \frac{16}{3}$. Plugging this into $b^2 = -a^2 + 12a$, we have $b^2 = -\left(\frac{16}{3}\right)^2 + 12\left(\frac{16}{3}\right) \to b^2 = \frac{320}{9} \to b = \pm\frac{\sqrt{320}}{3}$. Thus the point of intersection is $(\frac{16}{3}, \pm\frac{\sqrt{320}}{3})$.

The slope between this intersection and the original point $(12, 0)$ is $\frac{\pm\frac{\sqrt{320}}{3}}{\frac{16}{3}-12}$. This (very very ugly) fraction simplifies to $\pm\frac{2}{5}\sqrt{5}$. Through 'backwards rationalization', we can convert this into $\pm\frac{2}{\sqrt{5}}$. Thus, the equations that represent the line of sight are $\boxed{y = \pm\frac{2}{\sqrt{5}}(x - 12).}$

To find the locations of the y-axis that are not visible, we can find the values of y for which the equation is true when $x = 0$.

$$\begin{aligned} y &= \pm\frac{2}{\sqrt{5}}(x - 12) \\ &= \pm\frac{2}{\sqrt{5}}(-12) \\ &= \pm\frac{24}{\sqrt{5}} \end{aligned}$$

For some reason the textbook decides to rationalize an unrationalized expression here and unrationalize a rationalized expression in the above equation, but we will follow it regardless. Rationalizing $\pm\frac{24}{\sqrt{5}}$ yields $\pm\frac{24\sqrt{5}}{5}$. Thus, Dave cannot see in the range $\boxed{-\frac{24\sqrt{5}}{5} \leq y \leq \frac{24\sqrt{5}}{5}}$.

4.10 Problem: Pam is taking a train from the town of Rome to the town of Florence. Rome is located 30 miles due West of the town of Paris. Florence is 25 miles East, and 45 miles North of Rome. On her trip, how close does Pam get to Paris?

4.10 Solution: Let Rome be located at $(0, 0)$. Then, Florence would be located at $(25, 45)$ and Paris would be located at $(30, 0)$. Thus the line representing Pam's train's path would be $y = \frac{9}{5}x$. The shortest distance between Paris and this line can be represented with a perpendicular one, in this case with slope $-\frac{5}{9}$. Since this line extends from Paris to the line representing Pam's train's path, it goes through $(30, 0)$ (the location of Paris). Thus, the equation is $y = -\frac{5}{9}(x - 30)$. We want to find the intersection of these two lines.

$$\begin{aligned}
\frac{9}{5}x &= -\frac{5}{9}(x-30) \\
\frac{9}{5}x &= -\frac{5}{9}x + \frac{150}{9} \\
\frac{106}{45}x &= \frac{50}{3} \\
x &= \frac{50}{3} \cdot \frac{45}{106} = \frac{375}{53} \\
y &= \frac{9}{5} \cdot \frac{375}{53} = \frac{675}{53}
\end{aligned}$$

Thus, the intersection point is $(\frac{375}{53}, \frac{675}{53})$. The distance between this point and Paris, located at $(30, 0)$ is $\sqrt{\left(\frac{675}{53}\right)^2 + \left(\frac{375}{53} - 30\right)^2}$. Plugging this into a calculator yields about $\boxed{\text{26.2247 miles.}}$