Collingwood 15

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4.6a Context: The cup on the 9th hole of a golf course is located dead center in the middle of a circular green that is 70 feet in diameter. The ball follows a straight line path and exits the green at the right-most edge. Assume the ball travels a constant rate of 10 ft/sec.

4.6a Problem: Where does the ball enter the green?

4.6 Solution: Let the hole be the center of the coordinate system. Because the ball is 50 feet down and 40 feet left (in the diagram) of the hole, it is located initially at $(-40, -50)$. Since the green is 70 feet in diameter, it can be represented by the equation $x^2 + y^2 = 35^2 = 1225$. The rightmost location of the green is then $(35, 0)$. Since the ball travels from $(-40, -50)$ to $(35, 0)$, its slope is $\frac{0-(-50)}{35-(-40)} = \frac{50}{75} = \frac{2}{3}$ line can be represented by $y = \frac{2}{3}(x - 35)$. Simplifying, this comes to be $y = \frac{2}{3}x - -\frac{70}{3}$.

The ball first enters the green where x and y coordinates of the line representing the ball's trajectory satisfy the equation of the circle representing the green.

$$\begin{aligned}
x^2 + y^2 &= 1225 \\
x^2 + \left(\frac{2}{3}x - \frac{70}{3}\right)^2 &= 1225 \\
x^2 + \frac{4}{9}x^2 - 2 \cdot \frac{2}{3} \cdot \frac{70}{3}x + \frac{4900}{9} &= 1225 \\
\frac{13}{9}x^2 - \frac{280}{9}x + \frac{4900}{9} &= 1225 \\
\frac{13}{9}x^2 - \frac{280}{9}x - \frac{6125}{9} &= 0 \\
13x^2 - 280x - 6125 &= 0 \\
(13x + 175)(x - 35) &= 0
\end{aligned}$$

Hence, two solutions are $13x + 175 = 0 \to 13x = -175 \to x = -\frac{175}{13} \approx -13.46$ and $x - 35 = 0 \to x = 35$. Since we are looking for the location in which the ball *first* hits the green, the solution is the smaller value of x. Plugging in this value for y:

$$\begin{aligned}
y &= \frac{2}{3}x - \frac{70}{3} \\
y &= \frac{2}{3} \cdot -\frac{175}{13} - \frac{70}{3} \\
y &= -\frac{420}{13} \approx -32.31
\end{aligned}$$

Therefore, the ball enters the green at the location $\boxed{(-13.46, -32.31).}$

4.6b Problem: When does the ball enter the green?

4.6b Solution: The ball travels at 10 ft/sec. It begins at location $(-40, -50)$ and first enters the green at location $(-13.46, -32.31)$. Thus the distance can be calculated:

$$\begin{aligned} d &= \sqrt{(-40-(-13.46))^2+(-50-(-32.31)^2} \\ &= \sqrt{(-26.54)^2+(-17.69)^2} \\ &= \sqrt{1017.3077} \\ &\approx 31.8953 \end{aligned}$$

Thus, the ball must travel 31.8953 feet to enter the green. Thus it spends $\frac{31.8953}{10} \approx 3.19$ seconds. hence the ball takes $\boxed{\text{3.19 seconds}}$ to reach the green.

4.6c Problem: How long does the ball spend inside the green?

4.6c Solution: The ball exits the green at location $(35, 0)$, as specified by the problem. It enters the green at $(-13.46, -32.31)$. The distance spent in the green can be calculated between these two points:

$$\begin{aligned} d &= \sqrt{(35-(-13.46))^2+(0-(-32.31)^2} \\ &\approx 58.24 \end{aligned}$$

Because the ball travels at 10 ft/sec, it takes $\frac{58.24}{10} \approx 5.82$ seconds. Hence, the ball spends $\boxed{\text{5.82 seconds}}$ in the green.

4.6d Problem: Where is the ball located when it is closest to the cup and when does this occur.

4.6d Solution: The ball is closest to the cup when a line drawn from the current location of the ball to the location of the cup is perpendicular to the ball's trajectory. As this was represented by $y = \frac{2}{3}x - -\frac{70}{3}$, a line perpendicular to this passing through $(0, 0)$ (the location of the cup) would be $y = -\frac{3}{2}x$. We can find the intersection of these two lines to find the point when the ball is closest to the cup.

$$\begin{aligned} \frac{2}{3}x - \frac{70}{3} &= -\frac{3}{2}x \\ \frac{13}{6}x &= \frac{70}{3} \\ x &= \frac{140}{13} \approx 10.7692 \end{aligned}$$

Plugging this derived value of x to find the corresponding y-coordinate into the equation $y = -\frac{3}{2}x$ yields ≈ -16.154. Thus, the ball is closest the cup at $\boxed{\text{location } (10.77, -16.15)}$.

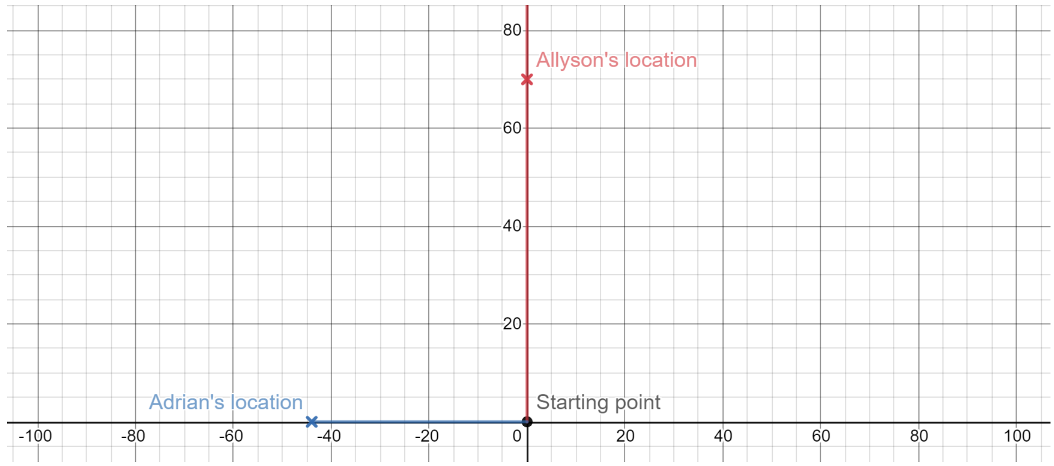
The ball travels to this point from $(-40, -50)$; thus it has moved 50.77 feet in the x-axis and 33.85 feet in the y-axis. Thus, the distance is $\sqrt{50.77^2 + 33.85^2} \approx 61.0197$ feet. Since the ball travels at 10 ft/sec, the ball is closest to the cup after about $\boxed{\text{6.1 seconds}}$.

4.7 Context: Allyson and Adrian have decided to connect their ankles with a bungee cord; one end is tied to each person's ankle. The cord is 30 feet long, but can stretch up to 90 feet. They both start from the same location. Allyson moves 10 ft/sec and Adrian moves 8 ft/sec in the directions indicated. Adrian stops moving at time $t = 5.5$ sec, but Allyson keeps on moving 10 ft/sec in the indicated direction.

4.7a Problem: Sketch an accurate picture of the situation at time $t = 7$ seconds. Make sure to label the locations of Allyson and Adrian; also, compute the length of the bungee cord at $t = 7$ seconds.

4.7a Solution: Let Allyson and Adrian begin at the origin (0, 0). Adrian moves in the negative x-direction, and Allyson moves in the positive y-direction. Adrian stops moving after 5.5 seconds, so we only need to compute his position then, which will be the same thereafter. Moving at 8 feet per second, he moves

$5.5 \cdot 8 = 44$ feet, landing at $\boxed{(-44, 0)}$. On the other hand, Allyson moves at 10 ft/sec, which means she travels 70 feet after 7 seconds to point $\boxed{(0, 70)}$. The distance between Adrian to the corner of the building at $(-20, 30)$ added to the distance from the corner of the building to Allyson is the length of the bungee cord. This is $\sqrt{(-44+20)^2 + (-30)^2} + \sqrt{(20)^2 + (70-30)^2}$, or about $\boxed{\text{83.14 feet.}}$



4.7b Problem: Where is Allyson when the bungee reaches its maximum length?

4.7b Solution: The line remains fixed in passing through the point $(-44, 0)$ (Adrian's location). The distance between Adrian and Allyson is interrupted by the corner of the building at $(-20, 30)$. Thus there are two components to distance: the distance between Adrian's position $(-44, 0)$ and the building corner $(-20, 30)$ and the distance between the building corner and Allyson's location. Let Allyson's y-coordinate be q.

$$\begin{aligned}
90 &= \sqrt{24^2 + 30^2} + \sqrt{20^2 + (30-q)^2} \\
90 - 38.4187 &= \sqrt{20^2 + (30-q)^2} \\
2660.70274 &= 20^2 + (30-q)^2 \\
2260.70274 &= (30-q)^2 \\
\pm 47.5468 &= 30 - q \\
\pm 47.5468 - 30 &= -q \\
q &= \pm 47.5468 + 30
\end{aligned}$$

Since Allyson's y-coordinate cannot be negative, she is at location $\boxed{(0,\ 77.54)}$.