Collingwood 13

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4.1 Context: This exercise emphasizes the "mechanical aspects" of working with linear equations. Find the equation of a line:

4.1a Problem: Passing through the points $(1, -1)$ and $(-2, 4)$.

4.1a Solution: The slope of a line that passes through $(1, -1)$ and $(-2, 4)$ is $\frac{5}{-3} = -\frac{5}{3}$. Solving for the y-intercept:

$$\begin{aligned} y &= -\frac{5}{3}x + b \\ -1 &= -\frac{5}{3}(1) + b \\ \frac{2}{3} &= b \end{aligned}$$

Thus, the equation for the line is $\boxed{y = -\frac{5}{3}x + \frac{2}{3}}$.

4.1b Problem: Passing through the point $(-1, -2)$ with slope $m = 40$.

4.1b Solution: An line with a slope of 40 can be represented with $y = 40x + b$. Deriving b:

$$\begin{aligned} y &= 40x + b \\ -2 &= -40 + b \\ 38 &= b \end{aligned}$$

Thus, the equation for the line is $\boxed{y = 40x + 38}$.

4.1c Problem: With y-intercept $b = -2$ and slope $m = -2$.

4.1c Solution: This line can be represented with $\boxed{y = -2x - 2}$.

4.1d Problem: Passing through the point (4,11) and having slope $m = 0$.

4.1d Solution: If a line has a slope of zero, then it is simply a horizontal line that satisfies $y = c$, where c is some constant. Since the line passes through the point (4, 11), which has a y-value of 11, the line is $\boxed{y = 11}$.

4.1e Problem: Perpendicular to the line in (a) and passing through (1,1).

4.1e Solution: The line in problem (a) has a slope of $-\frac{5}{3}$. Since the slope of a line perpendicular to another line of slope m is $-\frac{1}{m}$, or the negative reciprocal, the slope of a line perpendicular to the one in problem (a) has a slope of $\frac{3}{5}$. Thus, it can be represented with equation $y = \frac{3}{5}x + b$. Finding the appropriate value for b:

$$y = \frac{3}{5} + b$$
$$1 = \frac{3}{5} + b$$
$$\frac{2}{5} = b$$

Thus, the equation of the line is $\boxed{y = \frac{3}{5}x + \frac{2}{5}}$.

4.1f Problem: Parallel to the line in (b) and having y-intercept $b = -14$.

4.1f Solution: Lines that are parallel have the same slope. Thus, this line has a slope of 40. Since the y-intercept is given, the equation is $\boxed{y = 40x - 14}$.

4.1g Problem: Having the equation 3x + 4y = 7.

4.1g Solution: Making algebraic manipulations, we can derive slope-intercept form.

$$3x + 4y = 7$$
$$4y = -3x + 7$$
$$y = -\frac{3}{4} + \frac{7}{4}$$

Thus, the equation is $\boxed{y = -\frac{3}{4} + \frac{7}{4}}$.

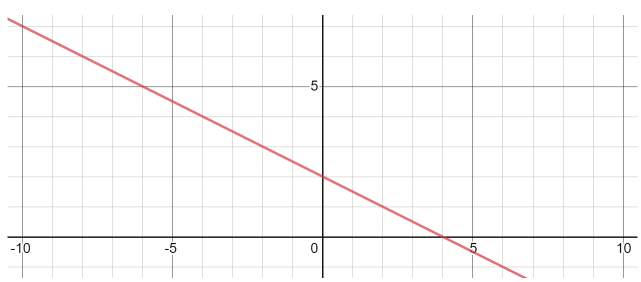
4.1h Problem: Crossing the x-axis at $x = 1$ and having slope $m = 1$.

4.1h Solution: If a line crosses the x-axis at $x = 1$, then it crosses the point (1, 0). Since the line has a slope of 1, it can be written as $y = x + b$. Deriving b:

$$y = x + b$$
$$0 = 1 + b$$
$$-1 = b$$

Thus, the equation of the line is $\boxed{y = x - 1}$.

4.2 Context: Sketch an accurate picture of the line having equation $y = 2 - \frac{1}{2}x$. Let α be an unknown constant.



4.2a Problem: Find the point of intersection between the line you have graphed and the line $y = 1 + \alpha x$; your answer will be a point in the xy plane whose coordinates involve the unknown α.

4.2a Solution: If the two lines are equal to each other, they must have the same y-coordinate. Setting them equal to each other yields the value of x in terms of α:

$$\begin{aligned}2 - \frac{1}{2}x &= 1 + \alpha x \\ (-\frac{1}{2} - \alpha)x &= -1 \\ x &= \frac{-1}{-\frac{1}{2} - \alpha} \\ x &= \frac{2}{1+2\alpha}\end{aligned}$$

Plugging this value into the simpler of the two equations:

$$\begin{aligned}y &= 1 + \alpha x \\ y &= 1 + \alpha(\frac{2}{1+2\alpha}) \\ y &= 1 + \frac{2\alpha}{1+2\alpha} \\ y &= \frac{1+2\alpha}{1+2\alpha} + \frac{2\alpha}{1+2\alpha} \\ y &= \frac{1+4\alpha}{1+2\alpha}\end{aligned}$$

Therefore, the point of intersection is $\boxed{(\frac{2}{1+2\alpha}, \frac{1+4\alpha}{1+2\alpha})}$.

4.1b Problem: Find α so that the intersection point in (a) has x-coordinate 10.

4.1b Solution: In order for the intersection to have an x value of 10, when we set up the equation to find the point of intersection, as in the previous problem $2 - \frac{1}{2}x = 1 + \alpha x$, we plug in the known value of x, which is 10. Thus, we can fill in variables to find the value of α:

$$\begin{aligned}2 - \frac{1}{2}x &= 1 + \alpha x \\ 2 - \frac{1}{2}(10) &= 1 + \alpha(10) \\ -4 &= 10\alpha \\ \alpha &= -\frac{2}{5}\end{aligned}$$

Therefore, $\boxed{\alpha = -\frac{2}{5}}$.

4.1c Problem: Find α so that the intersection point in (a) lies on the x-axis.

4.1c Solution: If the intersection point lies on the x-axis, then y must equal 0. Let us rewrite both equations:

$$\begin{aligned}y &= 2 - \frac{1}{2}x \\ 2y &= 4 - x \\ x &= -2y + 4\end{aligned}$$

$$
\begin{aligned}
y &= 1 + \alpha x \\
\alpha x &= y - 1 \\
x &= \frac{y}{\alpha} - \frac{1}{\alpha}
\end{aligned}
$$

Since the x values must be equal in an intersection point, we can set the two equations each to each other:

$$
\begin{aligned}
-2y + 4 &= \frac{y}{\alpha} - \frac{1}{\alpha} \\
-2(0) + 4 &= \frac{0}{\alpha} - \frac{1}{\alpha} \\
4 &= -\frac{1}{\alpha} \\
4\alpha &= -1 \\
\alpha &= -\frac{1}{4}
\end{aligned}
$$

Thus, $\boxed{\alpha = -\frac{1}{4}.}$