Collingwood 10

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3.4 Context: An amusement park Ferris Wheel has a radius of 60 feet. The center of the wheel is mounted on a tower 62 feet above the ground (see picture). For these questions, the wheel is not turning.

3.4a Problem: Impose a coordinate system.

3.4a Solution: Let the ground represent the x-axis, and let the point (0,0) represent the point where the center pipe of the Ferris wheel touches the ground (the 'tower'). The radius of the wheel is 60 feet, and it is held 2 feet above the ground by the tower. Thus, the top of the wheel is at (0, 122) and the Ferris wheel rotates around the point (0, 62). The equation of the circle is thus $x^2 + (y - 62)^2 = 3600$. The operator is located at (-24, 0).

3.4b Problem: Suppose a rider is located at the point in the picture, 100 feet above the ground. If the rider drops an ice cream cone straight down, where will it land on the ground?

3.4b Solution: We simply need to find the current x-coordinate of the rider; the ice cream will drop vertically at the same x-coordinate with y-coordinate value 0 (since it is at the ground). Since the rider is 100 feet high, we can plug this into the circle equation to find x:

$$\begin{aligned} x^2 + (y - 62)^2 &= 3600 \\ x^2 + (100 - 62)^2 &= 3600 \\ x^2 + (68)^2 &= 3600 \\ x^2 &= 2156 \\ x &= \pm 46.4327 \end{aligned}$$

Because the rider is depicted in the diagram as being right of the tower, the ice cream cone dropped by the rider will land $\boxed{\approx 46.43 \text{ feet right of the tower.}}$

3.4c Problem: The ride operator is standing 24 feet to one side of the support tower on the level ground at the location in the picture. Determine the location(s) of a rider on the Ferris Wheel so that a dropped ice cream cone lands on the operator. (Note: There are two answers.)

3.4c Solution: Ignoring the malicious nature of the problem, the ride operator is located at point (-24, 0). Thus, we need to determine the values of y for which the x is equal to -24.

$$\begin{aligned} x^2 + (y - 62)^2 &= 3600 \\ (-24)^2 + (y - 62)^2 &= 3600 \\ (y - 62)^2 &= 3024 \\ y - 62 &= \pm\sqrt{3024} \\ y &= \pm\sqrt{3024} + 62 \\ y &\approx 116.99, 7.01 \end{aligned}$$

Thus, the rider can be roughly at locations $\boxed{\text{(-24,117) and (-24, 7)}}$ to drop the ice cream cone onto the operator's head.

3.5 Context: A crawling tractor sprinkler is located as pictured below, 100 feet South of a sidewalk. Once the water is turned on, the sprinkler waters a circular disc of radius 20 feet and moves North along the hose at the rate $\frac{1}{2}$ inch/second. The hose is perpendicular to the 10 ft. wide sidewalk. Assume there is grass on both sides of the sidewalk.

3.5a Problem: Impose a coordinate system. Describe the initial coordinates of the sprinkler and find equations of the lines forming the North and South boundaries of the sidewalk.

3.5a Solution: Let the x-axis be the southern boundary of the sidewalk, and let (0,0) be the location where the perpendicular trajectory of the sprinkler touches (or shoots past) the road. Thus, the sprinkler begins at location (0, -100). The equation for the South boundary of the sidewalk is $\boxed{y = 0}$ and the equation for the North boundary of the sidewalk is $\boxed{y = 10}$.

3.5b Problem: When will the water first strike the sidewalk?

3.5b Solution: Initially, the water reaches, at its north-most, to (0, -80), since the radius of the water spray is 20 feet. The tractor moves north along $x = 0$ at a rate of $\frac{1}{2}$ inches per second, or 30 inches per minute. This is equivalent to $\frac{30}{12} = \frac{5}{2}$ feet per minute. At this rate, it will take the tractor's northernmost water reach $80 \cdot \frac{2}{5} = 32$ minutes to reach $x = 0$, the southern boundary of the sidewalk. Therefore, it takes $\boxed{\text{32 minutes}}$ for the water to first strike the sidewalk.

3.5c Problem: When will the water from the sprinkler fall completely North of the sidewalk?

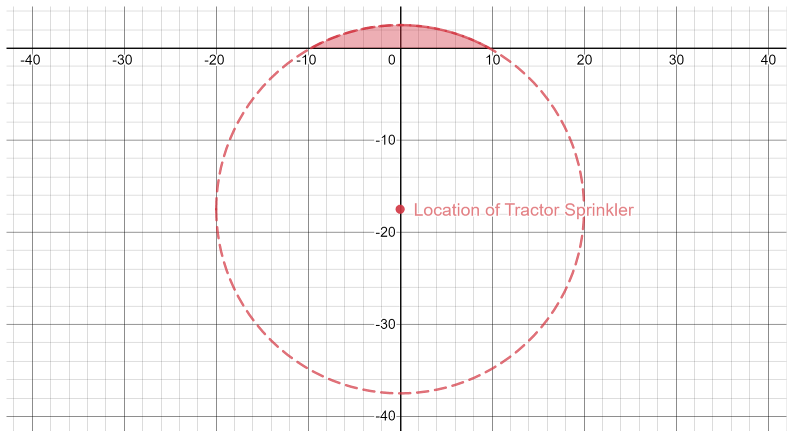
3.5c Solution: Initially, the southernmost reach of the water is (0, -120). For all the water in the sprinkler to fall North of the sidewalk, the southernmost reach must be at (0, 10). Thus, the tractor needs to travel 130 feet, which it can do in $130 \cdot \frac{2}{5} = 52$ minutes given the calculated rate of $\frac{5}{2}$ feet per minute (from 3.5b solution). Therefore, it takes $\boxed{\text{52 minutes}}$ for the water from the sprinkler to fall completely North of the sidewalk.

3.5d Problem: Find the total amount of time water from the sprinkler falls on the sidewalk.

3.5d Solution: To solve this, we must find the time t_s during which water from the tractor *starts* pouring onto the sidewalk and the time t_e during which water *ends* pouring onto the sidewalk. We can calculate this using the two times from 3.3 and 3.4. At $t_s = 32$ minutes, the northernmost reach of the water hits the sidewalk. Until $t_e = 52$ minutes, the water continues pouring onto the sidewalk. The duration of this period is $52 - 32 = 20$ minutes. Therefore, water from the sprinkler falls onto the sidewalk for $\boxed{\text{20 minutes.}}$

3.5e Problem: Sketch a picture of the situation after 33 minutes. Draw an accurate picture of the watered portion of the sidewalk.

3.5e Solution: The tractor, travelling at a rate of $\frac{5}{2}$ feet per minute, travels for 33 minutes. Thus, the tractor travels $33 \cdot \frac{5}{2} = 82.5$ feet. The tractor then lands at (0, -17.5). Visualization:



3.5f Problem: Find the area of GRASS watered after one hour.

3.5f Solution: The area of grass A_g watered after one area is equal to the total area of land A_l covered by the tractor, minus the watered area of the sidewalk A_s. A_g has three components: the bottom semicircle that is formed by the initial location of the tractor, a rectangle the width of the diameter and height determined by the initial and ending y-values of the tractor's center, and a top semicircle formed by the ending location of the tractor.

We can find the areas of these three components.

- The bottom and top semicircles form a complete circle. The radius of the circle is 20; thus the area is $20^2\pi = 400\pi \approx 1256.64$ square feet.
- The rectangle has width 40 (the diameter). It travels for one hour, moving at a rate of $\frac{5}{2}$ feet per minute; thus in one hour it travels $60 \cdot \frac{5}{2} = 150$ feet. The area of the rectangle is thus $150 \cdot 40 = 6000$ square feet.

Combining these areas yields 7256.64 square feet ($A_l = 7256.64$). From previous answers, we know that the tractor completely passes the road; thus we can represent the area watered as a rectangle where the width is the diameter of the sprinkler's water range and the height is the length of the road. These come to be 20 feet and 10 feet, respectively, and thus the area of the sidewalk watered is $40 \cdot 10 = 400$ square feet ($A_s = 400$).

Thus, the $\boxed{\text{area of grass watered is } 7256.64 - 400 \approx 6856.6 \text{ square feet.}}$