Collingwood Homework 1

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1.1.a Question: Verify that 7685.33 seconds is 2 hours, 8 minutes, and 5.33 seconds.

1.1.a Solution: Let us convert 2 hours, 8 minutes, and 5.33 seconds into seconds. There are 60 seconds in a minute, so there are $60 \cdot 8 = 480$ seconds in 8 minutes. There are 60 minutes in an hour, so there are $2 \cdot 60 \cdot 60 = 7,200$ seconds in 2 hours. Summing the total number of seconds for the three parts yields $7,200 + 480 + 5.33 = 7,685.33$ seconds. Hence, the two are $\boxed{\text{equal.}}$

1.1.b Question: Which is faster - 100mph or 150 ft/s?

1.1.b Solution: To truly compare the two rates, we must first convert them into the same units. We will convert 150 ft/s to miles per hour. Since there are 60 seconds in a minute and 60 minutes in an hour, there are $60 \cdot 60 = 3,600$ seconds in an hour. Hence, someone that travels 150 feet in a second will travel $150 \text{ feet} \cdot 3600 \text{ seconds} = 540,000$ feet in an hour.

Additionally, there are 5280 feet in a mile. Therefore, someone that travels 540000 feet in an hour will travel $\frac{540{,}000}{5{,}280} = 102.\overline{27}$ miles per hour. This is faster than 100 miles per hour.

Therefore, $\boxed{\text{150 ft/s is faster than 100 miles per hour}}$.

1.1.c Question: Gina's salary is 1 cent/second for a 40 hour work week. Tiare's salary is $1400 for a 40 hour work week. Who has a higher salary?

1.1.c Solution: To compare the two salaries, let us consider how much both make per hour.

- *Gina* makes 1 cent per second and works 40 hours during the week (this is irrelevant). Since there are 60 seconds in a minute and 60 minutes in an hour, there are $60 \cdot 60 = 3,600$ seconds in an hour. Since Gina makes 1 cent per second, she makes $3,600 \cdot 1 = 3,600$ cents per hour. Because there are 100 cents in a dollar, Gina makes $\frac{3{,}600}{100} = 36$ dollars per hour.
- *Tiare* makes $1,400 for a 40 hour work week. This means that she earns $\frac{1{,}400}{40} = 35$ dollars per hour.

Since $\$36 > \$35$, $\boxed{\text{Gina has the higher salary.}}$

1.1.d Question: Suppose it takes 180 credits to get a baccalaureate degree. You accumulate credit at the rate of one credit per quarter for each hour that the class meets per week. For instance, a class that meets three hours each week of the quarter will count for three credits. In addition, suppose that you spend 2.5 hours of study outside of class for each hour in class. A quarter is 10 weeks long. How many total hours, including times spent in class and time spent studying out of class, must you invest to get a degree?

1.1.d Solution: Since there are 10 weeks in a quarter, and to receive 180 credits we must enroll in classes that meet for 180 hours per week in total, we spend $10 \cdot 180 = 1,800$ hours in class to receive the required credits. However, since we also study for 2.5 hours for each hour of class, we spend an additional $2.5 \cdot 1,800 = 4,500$ hours. In total, we spend $\boxed{\text{6,300 hours}}$.

1.2 Question: Sarah can bicycle a loop around the north part of Lake Washington in 2 hours and 40 minutes. If she could increase her average speed by 1km/hour, it would reduce her time around the loop by 6 minutes. How many kilometers long is the loop?

1.2 Solution: It is worth framing the question in a $d = rt$ framework.

$$\textbf{a)}\ d \text{ kilometers} = r \tfrac{\text{kilometers}}{\text{hours}} \cdot \frac{(2 \cdot 60 + 40 \text{ minutes})}{60 \text{ minutes in an hour}}$$

Simplifying, we obtain:

$$\textbf{a)}\ d \text{ kilometers} = r \tfrac{\text{kilometers}}{\text{hours}} \cdot \frac{8}{3} \text{ hours}$$

If Sarah were to increase her speed by 1 km/hour, the following equation is true:

$$\textbf{b)}\ d \text{ kilometers} = (r+1) \tfrac{\text{kilometers}}{\text{hours}} \cdot \frac{(160 - 6)\text{minutes}}{60 \text{ minutes in an hour}}$$

Simplifying, we obtain:

$$\textbf{b)}\ d \text{ kilometers} = (r+1) \tfrac{\text{kilometers}}{\text{hours}} \cdot \frac{77}{30}\text{hours}$$

Setting the two equations equal to each other eliminates one variable, d.

$$\textbf{a)}\ \frac{8}{3}r = \textbf{b)}\ \frac{77}{30}(r+1)$$

$$\frac{8}{3}r = \frac{77}{30}r + \frac{77}{30}$$

$$\frac{1}{10}r = \frac{77}{30}$$

$$r = \frac{77}{3} \tfrac{\text{kilometers}}{\text{hours}}$$

Plugging this value of r into the first equation:

$$\textbf{a)}\ d \text{ kilometers} = \frac{77}{3} \tfrac{\text{kilometers}}{\text{hours}} \cdot \frac{8}{3} \text{ hours}$$

$$d = \frac{616}{9} \text{ kilometers}$$

Therefore, the loop is $\boxed{\tfrac{616}{9} \approx 68.4 \text{ kilometers.}}$